Graphs – Q4 [17 marks](25/5/21)

Exam Boards

OCR : -

MEI: -

AQA: Pure (Year 1)

Edx: -

(i) Sketch the curve $y = \frac{4x^2+5x+7}{2x+3}$ [9 marks]

(ii) Without using calculus, find the coordinates of the stationary points (to 3sf) [8 marks]

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Solution

(i) Step 1: $x = 0 \Rightarrow y = \frac{7}{3}$; [1 mark]

$4x^2 + 5x + 7 = 4(x + \frac{5}{8})^2 - \frac{25}{16} + 7 > 0$, so that there are no intersections with the x-axis [1 mark]

Step 2: vertical asymptote when $2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$ [1 mark]

$x = -\frac{3}{2} + \delta$ ($\delta > 0$ is small) $\Rightarrow y = \frac{+}{+}$; ie $y > 0$ [1 mark]

[$x = -\frac{3}{2} - \delta \Rightarrow y = \frac{+}{-}$; ie $y < 0$]

Step 3: To find $\lim_{x\to\infty} \frac{4x^2+5x+7}{2x+3}$:

$$\frac{4x^2+5x+7}{2x+3} = \frac{4x^2+6x}{2x+3} + \frac{-x+7}{2x+3} = 2x + \frac{-x+\frac{3}{2}}{2x+3} + \frac{-\frac{3}{2}+7}{2x+3}$$

$$= 2x - \frac{1}{2} + \frac{11}{2(2x+3)} \quad (*)$$

So $\lim_{x\to\infty} \frac{4x^2+5x+7}{2x+3} = 2x - \frac{1}{2}$

ie there is an oblique asymptote of $y = 2x - \frac{1}{2}$ [2 marks]

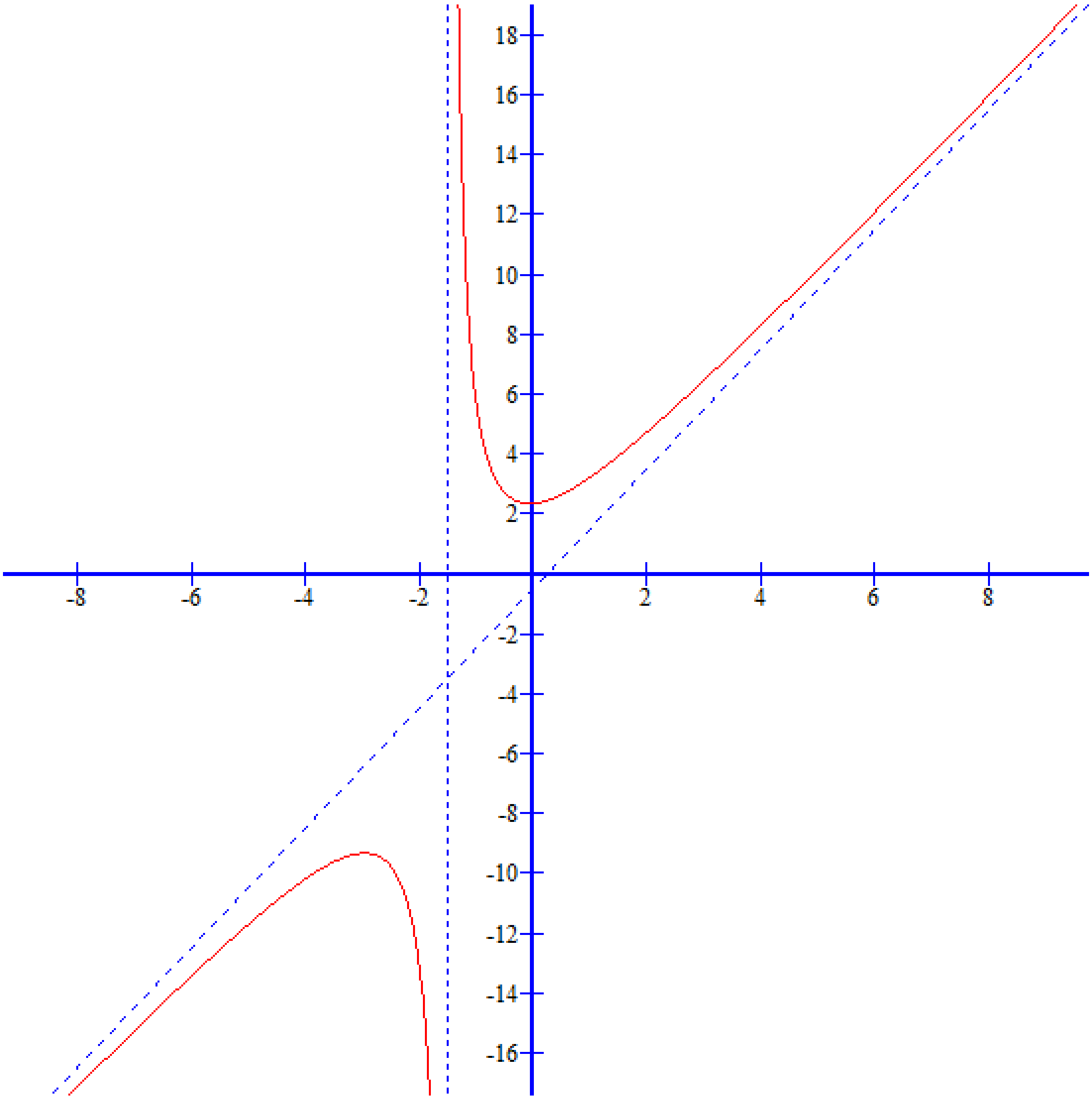
(also approached as $x \to -\infty$)

and, from (*), as $x \to \infty, y > 2x - \frac{1}{2}$, and as $x \to -\infty, y < 2x - \frac{1}{2}$

[1 mark]

[Note: We can't say $\lim_{x\to\infty}\frac{4x^2+5x+7}{2x+3}=\lim_{x\to\infty}\frac{4x+5+\frac{7}{x}}{2+\frac{3}{x}}=\frac{4x+5}{2}=2x+\frac{5}{2}$

as $lim\frac{f(x)}{g(x)}=\frac{lim\ f(x)}{lim\ g(x)}$ only when lim $f(x)$ and lim $g(x)$ are constants.]



[2 marks]

(ii) To find the stationary points, consider the values of x for which $\frac{4x^2+5x+7}{2x+3}=k$ has repeated roots; [1 mark]

Then $4x^2 + 5x + 7 = k(2x + 3)$

and $4x^2 + x(5 - 2k) + 7 - 3k = 0$, [1 mark]

with repeated roots occurring when the discriminant is zero,

so that $(5 - 2k)^2 - 16(7 - 3k) = 0$ [1 mark]

$\Rightarrow 4k^2 + k(-20 + 48) + 25 - 112 = 0$

ie $4k^2 + 28k - 87 = 0$

$\Rightarrow k = \frac{-28 \pm \sqrt{28^2 - 4(4)(-87)}}{8} = 2.33095$ or -9.33095 [2 marks]

The corresponding x-coordinates are $\frac{-(5-2k)}{8}$;

ie -0.042263 and -2.95774 [2 marks]

So there is a local minimum at $(-0.0423, 2.33)$ and a local maximum at $(-2.96, -9.33)$ (3sf). [1 mark]