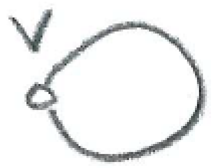
Graphs & Networks - Directed edges (5 pages; 19/11/19)

Contents

(1) Degree of a vertex in unusual cases

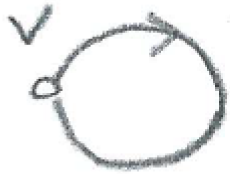
If a graph contains no directed edges, then the degree of a vertex is the number of edges incident to it



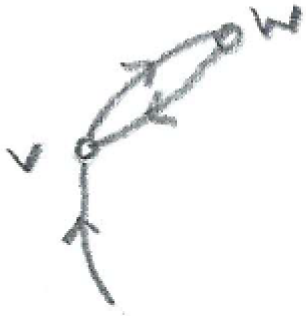
For the above diagram, the degree of V is 2. (From V's point of view, there are 2 edges leading away from it - it just so happens that they both lead back to V.)

If a graph contains only directed edges, then the number of incoming edges is referred to as the **indegree** of the vertex, and the number of outgoing edges is its **outdegree.** The total of the two is the degree of the vertex.

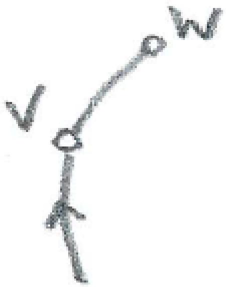
For the above diagram, the indegree of V is 1, the outdegree is 2, and the degree is 3.



For the above diagram, the indegree of V is 1, the outdegree is 1, and the degree is 2.



For the above diagram, the indegree of V is 2, the outdegree is 1, and the degree is 3.



A **mixed** graph is one with a mixture of directed and undirected edges (as in the above diagram), and the ideas of indegree and outdegree don't work as well: the edge VW could be described as both incoming and outgoing, but this would lead to a degree of 3 for V. However, from the point of view of the Route Inspection problem, the degree could usefully be thought of as 2 (ie of even order), but with a restriction on the direction of the route as it passes through V.

(2) Incidence (or adjacency) matrix

This shows the number of edges between the vertices of a graph.

(i) If there is no edge between two particular vertices, then this is recorded as a 0 (rather than a $-$, for example).

(ii) Graphs containing no directed edges

(a) A loop at vertex V would give rise to the entry 2 for (V,V) - as V could be left in either direction round the loop.

(b) The matrix will be symmetric.

(c) The degree of a vertex will equal the sum of the entries in its row (or column).

(iii) Graphs containing only directed edges

(a) An edge leading from A to B would give rise to the entry 1 for (A,B) and 0 for (B,A).

(b) A loop at vertex V would give rise to the entry 1 for (V,V)

[Note however that the contribution to the degree of V is 2 - see (1).]

(and n loops would give rise to the entry n)

(c) The matrix will not normally be symmetric (an exception being when each pair of vertices is joined by two edges with opposite directions).

(iv) Mixed graphs (containing both directed and undirected edges)

(a) A directed loop at vertex V would give rise to the entry 1 for (V,V).

(b) An undirected loop at vertex V would give rise to the entry 2 for (V,V) - as V could be left in either direction round the loop.

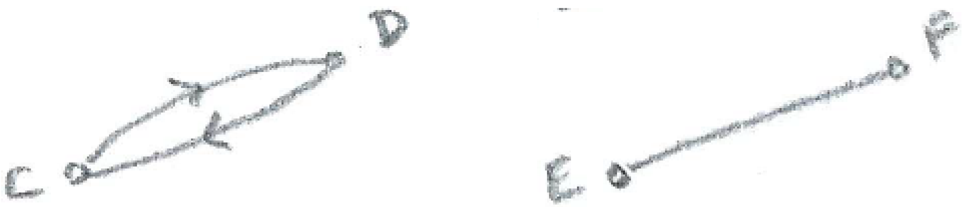
(c) A directed edge leading from A to B would give rise to the entry 1 for (A,B) and 0 for (B,A).

(d) The matrix will not normally be symmetric.

(e)

Referring to the diagram above, the entry for (A,B) would be 2, and for (B,A) it would be 1.

(f)



Referring to the diagrams above, note that the labelling of (C,D) as (1,1) and (D,C) as (1,1) would be the same as that of (E,F) and (F,E) (ie the incidence matrix could not distinguish between them).

(3) Matrix of weights (eg distance matrix)

This shows the weight of any edges **directly** linking the vertices of a network.

(i) If there is no edge between two particular vertices, then this is recorded as a $-$ (or sometimes an ∞). A 0 would indicate an edge of zero weight.

(ii) Networks containing no directed edges

(a) A single loop can be catered for.

(b) Multiple edges cannot be represented.

(c) The matrix will be symmetric.

(iii) Networks containing only directed edges

(a) One loop can be catered for.

(b) Double edges can be catered for, provided they have opposite directions.

(iv) Mixed networks (containing both directed and undirected edges)

(a) A single loop can be catered for (directed or undirected).

(b) Double edges can be catered for, provided they have opposite directions.

(c) Multiple undirected edges cannot be represented.

(d) Multiple edges involving both directed and undirected edges cannot be represented.