

Geometry – Q3 [Practice/E] (24/5/21)

Find the equation of the circle passing through the points

A (2,8) , B (7,3) and D (5,7)

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Solution

The first step is to find the centre of the circle, using the fact that the perpendicular bisector of each chord passes through the centre.

The chord AB has mid-point $(9/2, 11/2)$

and gradient $\frac{3-8}{7-2} = -1$

The perpendicular bisector of AB therefore has equation

$$\frac{y-11/2}{x-9/2} = -\frac{1}{-1}$$

$$\rightarrow 2y - 11 = 2x - 9$$

$$\rightarrow y = x + 1$$

The chord BD has mid-point (6, 5)

and gradient $\frac{7-3}{5-7} = -2$

The perpendicular bisector of BD therefore has equation

$$\frac{y-5}{x-6} = -\frac{1}{-2} \rightarrow y = \frac{1}{2}x + 2$$

The centre of the circle C is then found from the intersection of these lines:

$$x + 1 = \frac{1}{2}x + 2$$

so that $x = 2$ and $y = 3$

The radius is then the distance CA (for example)

$$= \sqrt{(2 - 2)^2 + (3 - 8)^2} = 5$$

Hence the equation of the circle is $(x - 2)^2 + (y - 3)^2 = 25$

(Check: B and D satisfy the equation.)