Geometry – Q1 [Practice/E] (23/5/21)

Show that the area of triangle ABC is given by

$$\frac{1}{2}\sqrt{|\overrightarrow{AB}|^2|\overrightarrow{AC}|^2 - (\overrightarrow{AB}.\overrightarrow{AC})^2}$$

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Solution

$$\frac{1}{2}\sqrt{|\overrightarrow{AB}|^2|\overrightarrow{AC}|^2 - (\overrightarrow{AB}.\overrightarrow{AC})^2} = \frac{1}{2}\sqrt{|\overrightarrow{AB}|^2|\overrightarrow{AC}|^2 - (|\overrightarrow{AB}||\overrightarrow{AC}|cosA)^2}$$

$$= \frac{1}{2}\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|\sqrt{1 - cos^2A} = \frac{1}{2}\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|SinA = \text{Area of triangle}$$