Forces - Notes (6 pages; 14/11/18)

(1) To find the resultant of two forces, it will usually be easier to resolve the forces first, and then add the components; rather than using the Cosine rule.

(2) Force Diagrams

The first step in many Mechanics problems is to create a force diagram for the main object of interest. The next step is to resolve forces in two (convenient) perpendicular directions, and apply Newton's 2nd law. A further equation can be obtained in the case of rigid bodies in equilibrium, by taking moments about a suitable point.

(3) Objects attached to, or resting against a wall (eg a ladder):

Usually it is best to resolve the components of the force applied by the wall to the object, along and perpendicular to the wall. In the case of a ladder, which isn't attached to the wall, the component along the wall is the frictional force, and this would be considered to be zero if the wall was smooth.

In the case of an object (such as a rod) that is attached to the wall, the component along the wall is a reaction force.

In situations involving only two other forces (eg the weight of the rod and the tension in a cable holding it in place), it may be advantageous to work with a single reaction force at the wall (at an - as yet - unknown angle), in order to make use of the fact that 3 forces in equilibrium (no two of which are parallel) are 'concurrent' (ie meet at a point).

Similarly, in the case of a block resting on a slope, the reaction force is usually resolved into the normal reaction (perpendicular to the slope) and the frictional force.

(4) Block on a slope: direction in which forces are resolved

The best directions in which to resolve forces will usually be along and perpendicular to the slope. This takes advantage of the fact that there will be no acceleration perpendicular to the slope - and this will be true whether or not the block is accelerating.

(If the block is not accelerating, then forces could be resolved vertically and horizontally.)

(5) Implication of a smooth pulley

A rope (of mass m) over a pulley has 3 external forces on it: the tension at the two ends (T_1 & T_2) and the frictional force (F) due to the pulley.

Applying Newton's 2nd law to the rope,

$T_1 - T_2 - F = ma$ (where a is the acceleration of the system)

It is usually convenient for T_1 & T_2 to be equal (in order for there not to be too many variables). This will be possible if the rope is assumed to have negligible mass, and if F is also negligible; ie if the pulley is smooth.

(6) Implication of a rope of negligible mass

See "Implication of a smooth pulley".

(7) Implication of an inextensible rope

This ensures that all components of a system have the same acceleration.

(8) Combining objects

In the case of a car pulling a trailer, joined by a towbar, it is possible to treat the car , trailer and towbar as a single object (so that the tension in the towbar is treated in the same way as any other internal force of the system).

In the case of a block on a table, connected via a pulley to another block which is falling down the side of the table, we cannot strictly speaking treat the two blocks and the connecting rope as a single object - as they are not moving in the same direction.

However, we can combine the equations of motion for the two blocks as follows:

$mg - T = ma$ & $T - F = Ma$

(where m is the mass of the falling block, and F is the frictional force on the block on the table, which has mass M)

Adding these equations gives: $mg - F = (m + M)a$

(9) Friction on a slope

Note that friction could be up or down the slope. An applied force may be just sufficient to stop an object from sliding down a slope, in which case friction will be opposing the attempted motion, which is down the slope; so that friction is up the slope - aiding the applied force. If instead the applied force is not quite enough to move the object up the slope, then the attempted motion is up the slope, and the friction is down the slope - countering the applied force). In both cases, the size of the limiting frictional force is the same.

(10) If an object is in equilibrium, then the total moment of the forces on it will be the same, whatever point it is taken about.

(11) To establish the moment of a force about a particular point, either:

(a) find the perpendicular distance, or

(b) resolve the force in two convenient perpendicular directions

[It can be shown that the moment doesn't depend on the point (on the line of action of the force) at which the force is resolved.]

(12) Equilibrium methods:

(i) Resolve forces in two perpendicular directions and take moments about a convenient point.

(Choose the point so as to exclude unwanted forces and/or reduce the number of forces appearing in the resulting equation.)

(ii) Take moments about two points, and resolve forces in one direction (provided that the direction is not perpendicular to the line joining the two points).

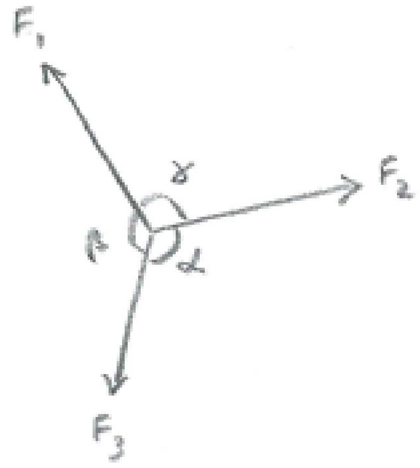
(iii) Take moments about 3 points (provided that they do not lie on a straight line).

(iv) In the case of 3 forces (with no forces being parallel): the lines of action must meet at a point (be 'concurrent').

(v) If the forces are represented by vector arrows, then they will form a vector polygon.

(vi) Lami's theorem (similar to the Sine rule)

$$\frac{F_1}{sin\alpha} = \frac{F_2}{sin\beta} = \frac{F_3}{sin\gamma}$$



(13) Implication of a "freely (or smoothly) hinged joint"

This means that there is no hidden moment about the joint. Consider a door with rusty hinges: there is a frictional force close to the point about which the door turns, and this provides a moment.

(14) Friction associated with a rolling or sliding wheel

[See "Rolling wheels"]

There are several possibly surprising results associated with rolling wheels.

(i) The point of a rolling wheel that is in contact with the ground is (momentarily) stationary. The contact point has two components of velocity: that of the centre of mass of the wheel, and a rotational velocity. These can be shown to cancel out.

The wheel is effectively toppling continually about the contact point.

(ii) Because the contact point is stationary, any friction that arises (apart from 'rolling friction' - see below) will be static friction, for a rolling wheel. If there is sliding, then dynamic friction applies instead.

(iii) If a wheel is not rolling at constant speed (ie if it is either accelerating or braking), then its acceleration has two components: translational and rotational. The translational acceleration is due to the combined effect of friction and the other resistance forces; ie not the driving force of the engine - although this gives rise to the friction.

(iv) If a car is accelerating because of a torque applied by the engine, and the wheels are rolling, then the friction (which opposes motion of the point of contact) is in the same direction as the motion of the car (and in fact provides the forward force on the car).

(v) If a cart is being pulled by a horse (or if a car is being pushed), and the wheels are rolling, then friction opposes the attempted forward motion of the point of contact (ie the attempt to drag it across the ground), and is therefore in the opposite direction to the motion of the cart or car; ie it is hindering the forward force being applied. The same applies when a ball is rolling down a slope (the component of the weight down the slope replaces the pulling or pushing force).

(vi) If a wheel is rolling at constant speed, then there is no friction at the contact point: As there is no angular acceleration, there is no force for the friction to oppose. Also, ignoring resistance forces (which would in practice slow the wheel down), any friction would be applying a translational force to the wheel (and this cannot be the case, as it is moving with constant speed).

(vii) Confusingly, the term 'rolling friction' is used, as part of the resistance forces on the wheel (others being air resistance and friction at the axle) for the resistance that occurs due to the way that a tyre compresses. This is separate from the static friction at the point of contact (in the case of rolling with acceleration) or dynamic friction (in the case of sliding).