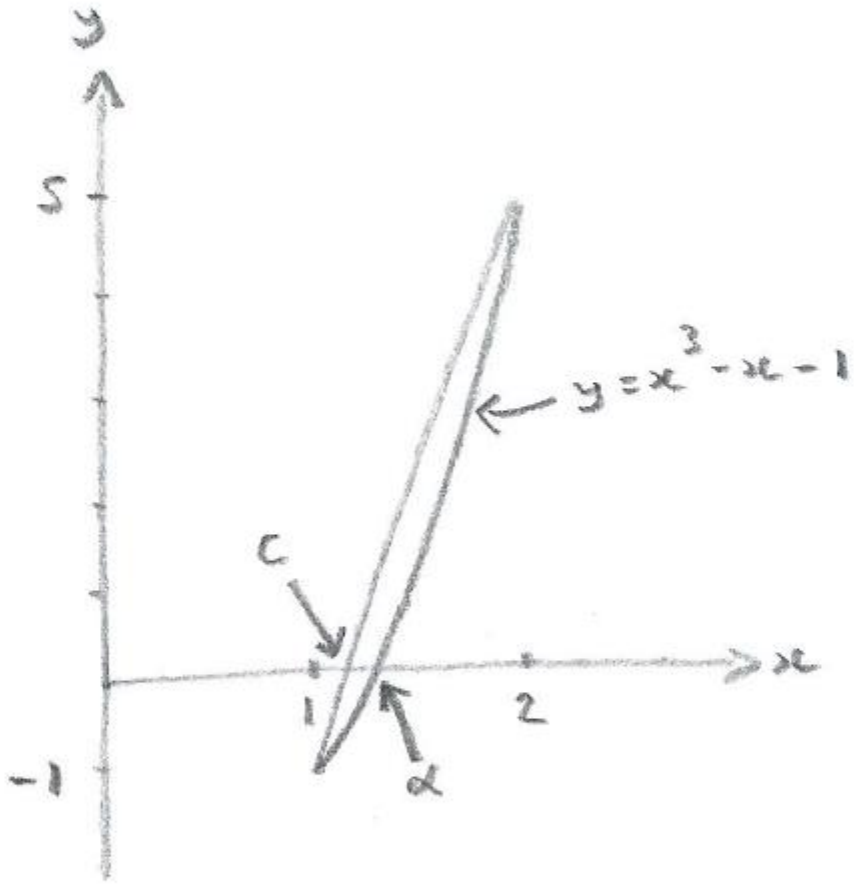
Numerical Solution of Equations - Method of False Position

(4 pages; 22/10/18)

[Also known as linear interpolation.]

(1) Example



To find the root α between 0 (a, in general) and 1 (b, in general) for $x^3 - x - 1 = 0$:

Step 1: $f(1) = -1$ & $f(2) = 5$; $f(\alpha) = 0$

Step 2: We construct a line between the points $(1, -1)$ and $(2, 5)$.

The first estimate of the root will be where this line crosses the

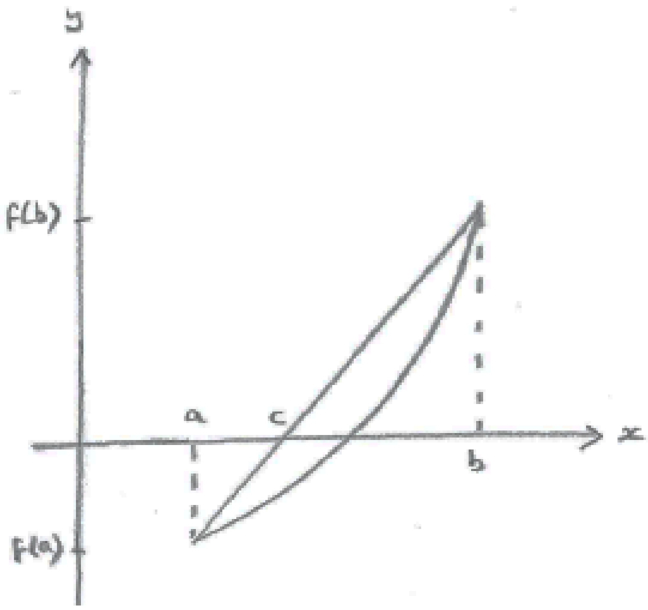
x-axis.

By linear interpolation, this estimate (c, say) will be the following weighted average of 1 and 2:

$c = \left(\frac{5}{6}\right)(1) + \left(\frac{1}{6}\right)(2)$, where $\frac{5}{6} = \frac{f(2)}{f(2)+[-f(1)]}$ and $\frac{1}{6} = \frac{[-f(1)]}{f(2)+[-f(1)]}$

[Note that the larger weight is being applied to 1, as c is nearer 1 than 2.]

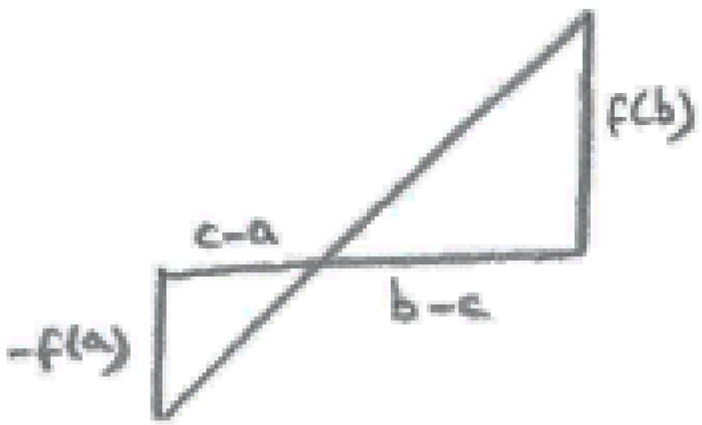
In general, $c = \left(\frac{f(b)}{f(b)-f(a)}\right)a + \left(\frac{-f(a)}{f(b)-f(a)}\right)b = \frac{af(b)-bf(a)}{f(b)-f(a)}$



Step 3: Establish the sign of $f(c)$. If $f(c)<0$ [as in this example], repeat the process for the interval (c, b). If $f(c) > 0$, repeat the process for the interval (a, c).

a	b	f(a)	f(b)	c	f(c)
1	2	-1	5	1.166667	-0.5787
1.166667	2	-0.5787	5	1.253112	-0.28536
1.253112	2	-0.28536	5	1.293437	-0.12954
1.293437	2	-0.12954	5	1.311281	-0.05659
1.311281	2	-0.05659	5	1.318989	-0.0243
1.318989	2	-0.0243	5	1.322283	-0.01036
1.322283	2	-0.01036	5	1.323684	-0.0044
1.323684	2	-0.0044	5	1.324279	-0.00187
1.324279	2	-0.00187	5	1.324532	-0.00079

(2) Similar triangles derivation



$$\frac{c-a}{-f(a)} = \frac{b-c}{f(b)}$$

$$\Rightarrow f(b)(c-a) = f(a)(c-b)$$

$$\Rightarrow c[f(b)-f(a)] = af(b) - bf(a)$$

$$\Rightarrow c = \frac{af(b)-bf(a)}{f(b)-f(a)}$$

(3) Equation of straight line derivation

Referring to the earlier diagram, the equation of the straight line is $\frac{y-f(a)}{x-a}=\frac{f(b)-f(a)}{b-a}$

Substituting the point $(c,0)$ gives $\frac{0-f(a)}{c-a}=\frac{f(b)-f(a)}{b-a}$

$\Rightarrow f(a)(a-b)=[f(b)-f(a)](c-a)$

$\Rightarrow a[f(a)+f(b)-f(a)]-bf(a)=[f(b)-f(a)]c$

$\Rightarrow c=\frac{af(b)-bf(a)}{f(b)-f(a)}$