

## A Level & Further Maths Topics by Exam Board - Pure (22 pages; 7/8/19)

### A Level

P: material common to AS and AL

P\*: material for 2nd year of AL only

### Further Maths

#### OCR

##### Pure Core

PC: material common to AS and AL

PC\*: material for 2nd year of AL only

##### Additional Pure

AP: material common to AS and AL

AP\*: material for 2nd year of AL only

## OCR B (MEI)

### Core Pure (CP)

CP: material common to AS and AL

CP\*: material for 2nd year of AL only

Extra Pure (EP\*) [not for AS]

Numerical Methods (N) [AS or AL]

[not included: Further Pure with Technology]

## AQA

### Pure

P: material common to AS and AL

P\*: material for 2nd year of AL only

Discrete [for Binary Operations & Groups only]

D: material common to AS and AL

D\*: material for 2nd year of AL only

Note: AQA specifications don't give any guidance, but there are useful notes for OCR, MEI & EDX, which are often relevant to AQA (or each other).

## **EDX**

### Core Pure

CP: material common to AS and AL

CP\*: material for 2nd year of AL only

### Further Pure 1

FP1: material common to AS and AL

FP1\*: material for 2nd year of AL only

### Further Pure 2

FP2: material common to AS and AL

FP2\*: material for 2nd year of AL only

### Decision 2 [Recurrence Relations only]

D2: material common to AS and AL

D2\*: material for 2nd year of AL only

	fmng reference Y ⇒ note exists	OCR	OCR B (MEI)	AQA	EDX
<b>Algebra</b>					
Law of indices		P	P	P	P
Manipulating surds		P	P	P	P
- rationalising the denominator		P	P	P	P
Simultaneous eq'ns					
- elimination		P	P	P	P
- substitution		P	P	P	P
- one linear, one quadratic		P	P	P	P
Simplify rational expressions		P*	P*	P*	P*
<b>Circles</b>	Circle theorems				
Equation		P	P	P	P
- completing the square		P	P	P	P
Angle in semicircle is right-angle		P	P	P	P
Perpendicular from centre bisects chord		P	P	P	P
Tangent is perpendicular to radius		P	P	P	P
Intersection of line & circle		P	P		
<b>Complex Numbers</b>	Y				

Introduction	P1	PC	CP	P	CP
Mod-arg form	P1	PC	CP	P	CP
Polynomial eq'ns	P2	PC	CP	P	CP
Loci	P2	PC	CP	P	CP,FP2
de Moivre's thm	P2	PC*	CP*	P*	CP*
Roots of complex nos	P2	PC*	CP*	P*	CP*
Transformations from z-plane to w-plane					FP2*
<b>Conics</b>					
Parabola	Y				
- sketching				P	
- cartesian & parametric eq'ns					FP1
- focus-directrix					FP1
- tangent & normal					FP1
- loci problems					FP1
Rect. hyperbola	Hyperbolas				
- sketching				P	
- cartesian & parametric eq'ns					FP1
- tangent & normal					FP1
- loci problems					FP1*
Ellipse	Y				
- sketching				P	

- cartesian & parametric eq'ns					FP1*
- focus-directrix; eccentricity					FP1*
- tangent & normal					FP1*
- loci problems					FP1*
General hyperbola	Y			P	
- sketching					
- cartesian & parametric eq'ns					FP1*
- focus-directrix; eccentricity					FP1*
- tangent & normal					FP1*
- loci problems					FP1*
<b>Differential Eq'ns</b>					
Constructing (1st order) differential eq'ns		P*	P*	P*	P*
Separation of variables		P*	P*	P*	P*
Integrating factor for 1st order eq'ns	Y	PC*	CP*	P*	CP*
2nd order linear eq'ns (constant coeffs)	Y	PC*	CP*	P*	CP*
SHM	Mechanics: DE - Oscillations	PC*	CP*	P*	CP*
Damped oscillations	Mechanics: DE - Oscillations	PC*	CP*	P*	CP*
Paired linear 1st order eq'ns	Y	PC*	CP*	P*	CP*

Euler's method of approximation	Approximate methods			P*	
Numerical sol'n of 1st & 2nd order eq'ns					FP1
Taylor series sol'n					FP1*
Use of given substitution					FP1*
<b>Differentiation</b>					
$f'(x)$ as gradient of tangent to curve, or rate of change		P	P	P	P
Sketching gradient f'n		P	P	P	P
2nd derivative as rate of change of gradient		P	P	P	P
- in relation to convex/concave sections of curve, and points of inflexion		P*	P*	P*	P*
Derivative of $x^n$ for rational $n$ , from 1st principles		P	P	P	P
Derivative of $\sin x$ & $\cos x$ , from 1st principles		P*	P*	P	P*
Derivative of $e^{kx}$		P*	P*	P*	P*
Derivative of $a^{kx}$		P*	P*	P*	P*
Derivative of $\sin kx$ , $\cos kx$ , $\tan kx$		P*	P*	P*	P*
Derivative of $\ln x$		P*	P*	P*	P*

Eq'ns of tangents & normals		P	P	P	P
Find & classify stationary points (using 1st or 2nd derivative)		P	P	P	P
Identify increasing/decreasing fns		P	P	P	P
Find points of inflexion		P*	P*		P*
Classify PoI as stationary or non-stationary		P*	P*		
Product & Quotient rule		P*	P*	P*	P*
Chain rule		P*	P*	P*	P*
- connected rates of change		P*	P*	P*	P*
$\frac{dx}{dy}$ is reciprocal of $\frac{dy}{dx}$		P*	P*	P*	P*
finding gradient in terms of a parameter		P*	P*	P*	P*
implicit differentiation		P*	P*	P*	P*
Leibnitz's thm					FP1*
L'hôpital's rule				P*	FP1*
<b>Exponential Function</b>					
Sketching $y = a^x$		P	P	P	P
Sketching $y = e^x$		P	P	P	P
Gradient of $e^{kx}$ is $ke^{kx}$ (for models)		P	P	P	P
Models of exponential growth and decay		P	P	P	P

<b>Functions</b>		P*	P*		
Mappings		P*	P*		
Inverse functions		P*	P*	P*	P*
Composite functions		P*	P*	P*	P*
<b>Graphs</b>					
Sketching cubics and quartics		P	P	P	P
Sketching $y = \frac{a}{x}$ & $y = \frac{a}{x^2}$		P	P	P	P
Graphical interpretation of sol'n of eq'n		P	P	P	P
Graphs of proportional relations		P	P	P	P
Graphs of transformed functions (translations, stretches, reflections) - single transformations		P	P	P	P
- combinations of transformations		P*	P*	P*	P*
- rotations & enlargements				P*	
Sketching rational functions	Y			P	
Range and stationary points of rational fns	Y			P	
reciprocals of functions				P*	
<b>Groups</b>	Groups P1&2				
Binary operations	not covered	AP		D	
Subgroups		AP	EP*	D*	FP2

Cyclic groups		AP	EP*	D*	FP2
Generators		AP		D*	
Lagrange's thm		AP*	EP*	D*	FP2
Isomorphisms		AP*	EP*	D*	FP2*
Abstract groups		AP*			
<b>Hyperbolic functions</b>	Y	PC*	CP*	P	CP*
Reciprocal functions				P*	
Inverse Hyperbolic functions		PC*	CP*	P	CP*
- log form		PC*	CP*	P	CP*
Derivatives & integrals of hyperbolic functions		PC*	CP*	P*	CP*
<b>Inequalities</b>	Y				
Linear & quadratic		P	P	P	P
Graphical approach		P	P	P	P
Set & interval notation		P	P	P	P
involving polynomial eq'ns				P	FP1
involving modulus functions				P*	FP1*
<b>Integration</b>					
Fundamental thm of Calculus		P	P	P	P
Integral of $x^n$ ( $n \neq -1$ )		P	P	P	P

Evaluating constant of integration	P	P		P	
Integral of $e^{kx}$	P*	P*	P*	P*	
Integral of $\sin x$ & $\cos x$	P*	P*	P*	P*	
Integral of $\frac{1}{x}$	P*	P*	P*	P*	
Definite integrals	P	P	P	P	
Area under curve	P	P	P	P	
Area between curves	P*	P*	P*	P*	
- curves may be defined parametrically	P*				
Integration as limit of a sum	P*	P*	P*	P*	
Integration by substitution	P*	P*	P*	P*	
Integration by Parts	P*	P*	P*	P*	
- involving 2 applications	P*	P*	P*	P*	
$\int \ln x \, dx$ by Parts	P*	P*			
Improper Integrals	Y	PC*	CP*	P*	CP*
- $\lim_{x \rightarrow \infty} x^k e^{-x}$ & $\lim_{x \rightarrow 0} x^k \ln x$	not covered			P*	
Volumes of revolution	PC*	CP*	P	CP	
- parametric form	PC*			CP*	
Mean value of function	PC*	CP*	P (sic)	CP*	
Use of partial fractions	PC*	CP*	P*	CP*	
Use of inverse trig. fns	PC*	CP*	P*	CP*	
Trig. substitutions	PC*	CP*	P*	CP*	
Use of inverse hyperbolic fns	PC*	CP*	P*	CP*	

Reduction formulae	Integration Methods	AP*		P*	FP2*
Hyperbolic substitutions			CP*	P*	CP*
Arc length & surface area	Y	AP*		P*	FP2*
Weierstrauss subst. (t formulae)	Integration Methods				FP1*
<b>Logarithms</b>		P	P	P	P
$\log_a x$ as inverse of $a^x$		P	P	P	P
$a = b^c \Leftrightarrow \log_b a = c$		P	P		
$\ln x$ as the inverse of $e^x$		P	P	P	P
Laws of logarithms [excl. change of base]		P	P	P	P
Solving $a^x = b$ or related inequality		P	P	P	P
Use logarithmic graphs to estimate parameters in $y = ax^n$ and $y = kb^x$ given data		P	P	P	P
<b>Matrices</b>		PC	CP	P	CP
Transformations	Y	PC	CP	P	CP
Invariant points & lines	Y	PC	CP	P	CP
2x2 det & inverse [eq'n's, transformations]	Y	PC	CP	P	CP

3x3 det (w/o calc)	Y	PC	CP*	P*	CP
inverse (w/o calc)	Y	PC	CP*	P*	CP
Simultaneous eq'ns - unique sol'n		PC	CP	P*	CP
Simultaneous eq'ns - non-unique sol'n		PC*	CP	P*	CP
Eigenvectors	Y		EP*	P*	FP2
Diagonalisation	Eigenvectors			P*	FP2
Cayley-Hamilton thm			EP*		FP2
Factorisation of dets				P*	
<b>Modulus function</b>		P*	P*	P*	P*
Solve eq'ns & inequalities involving modulus function		P*	P*		
- graphical approach		P*			
Sketching $y =  ax + b $		P*	P*	P*	P*
<b>Multivariable calculus</b>					
Partial differentiation		AP	EP*		
3D surfaces		AP	EP*		
Sections & contours		AP	EP*		
Stationary points		AP	EP*		
Hessian matrix		AP*			
Tangent planes		AP*	EP*		

<b>Number Theory</b>	Y				
Divisibility tests		AP			FP2
Division algorithm		AP			FP2
Bezout's identity					FP2
Properties of congruences					FP2
Solving linear congruences		AP			FP2*
Solving simultaneous linear congruences		AP*			
Prime numbers		AP			
Euclid's lemma		AP			
Fermat's Little theorem		AP*			FP2*
Order of $a \text{ mod } p$		AP*			
Binomial theorem	not covered	AP*			
Combinatorics	not covered				FP2*
<b>Numerical Methods</b>					
Use of spreadsheets & calculators	Calculator method		N		
<b>Errors</b>	Y				
Absolute & relative error			N		
Error propagation			N		
Error in the representation of numbers			N		

Order of convergence & order of method	Convergence - Introduction; Convergence - Sol'n of Eq'ns; Convergence - Numerical Integration		N		
Error analysis			N		
<b>Solution of equations</b>					
Change of Sign method		P*	P*	P*	P*
- verifying accuracy of an approximation		P*			
- failure of Change of Sign method		P*	P*	P*	P*
Bisection method	Y		N		
Method of False position (aka linear interpolation)	Y		N		
Secant method	Y		N		
Fixed point method ( $x = g(x)$ )	Y	P*	P*,N	P*	P*
- failure of method		P*	P*	P*	P*
Newton-Raphson method	Y	P*	P*,N	P*	P*
- failure of method		P*	P*	P*	P*
<b>Numerical Differentiation</b>	Y				
Forward Difference method			N		

Central Difference method			N		
<b>Numerical Integration</b>					
Rectangles		P*	P*		
Mid-ordinate (aka midpoint) rule	Midpoint		N	P*	
Trapezium rule	Y	P*	P*,N	P*	P*
- over or under-estimate		P*	P*		P*
Simpson's rule	Y		N	P*	FP1*
Relationship between methods			N		
<b>Approximation to function</b>					
Newton's Forward Difference method	Y		N		
Lagrange's method	Y		N		
<b>Parametric eq'ns</b>					
Converting between cartesian & parametric forms		P*	P*	P*	P*
Sketching parametric curves		P*			
<b>Partial fractions</b>	Y				
No more than squared linear terms in denom., and constant or linear term in num.		P*	P*	P*	P*
Quadratic denominator		PC*	CP*	P*	CP*

Numerator not having lower degree		PC*			
<b>Polar Coordinates</b>	Y				
Sketching curves		PC*	CP*	P (sic)	CP*
Convert between polar & cartesian		PC*	CP*	P (sic)	CP*
Area enclosed by curves		PC*	CP*	P*	CP*
<b>Polynomials</b>					
Factorising		P	P	P	P
Division		P	P	P	P
Factor Theorem		P	P	P	P
Relations between coeffs & roots	Y	PC	CP	P	CP
Transformation of eq'ns		PC	CP	P	CP
<b>Proof</b>					
Deduction		P	P	P	P
Exhaustion		P	P	P	P
Counter-example		P	P	P	P
Contradiction		P*	P*	P*	P*
Irrationality of $\sqrt{2}$		P*	P*	P*	P*
Infinity of primes		P*	P*	P*	P*
<b>Proof by Induction</b>	Y				

Sum of series		PC*	CP	P	CP
Recurrence relations		AP	CP		FP2
Powers of matrices		PC	CP	P	CP
Divisibility tests		PC	CP*	P	CP
<b>Quadratic Functions</b>		P	P	P	P
Completing the square		P	P	P	P
Roots of quadratic eq'n		P	P	P	P
<b>Recurrence Relations</b>	Y				
Sol'n of 1st order eq'ns					D2
Sol'n of 2nd order eq'ns					D2*
<b>Sequences</b>		P*	P*	P*	P*
Inductive & deductive formulae		P*	P*	P*	P*
convergence & divergence			P*		
Fibonacci numbers		AP			
1st order recurrence relations	Y	AP	EP*		FP2
2nd order recurrence relations	Y	AP*	EP*		FP2*
<b>Series</b>					
Binomial expansion - positive integer power		P	P	P	P

link to Binomial probabilities		P	P	P	P
General Binomial expansion		P*	P*	P*	P*
Range of validity		P*	P*	P*	P*
Sigma notation		P*	P*	P*	P*
Arithmetic series - sum to n terms (incl. proof)		P*	P*	P*	P*
Geometric series - sum to n terms (incl. proof)		P*	P*	P*	P*
- sum to infinity (incl. proof)		P*	P*	P*	P*
Formulae for sums of $r, r^2 \text{ & } r^3$		PC*	CP	P	CP
Method of differences	Y	PC*	CP	P	CP*
Maclaurin Series	Y	PC*	CP*	P	CP*
Taylor series	Y				
Use of series to find limits				P*	FP1*
<b>Straight lines</b>					
Equation		P	P	P	P
Perpendicular lines		P	P	P	P
<b>Trigonometry</b>	Y				
Sine, cos & tan of any angle		P	P	P	P
- reference to unit circle			P		

Sine & Cosine rules	P	P	P	P
Area of triangle: $\frac{1}{2}ab\sin C$	P	P	P	P
Radians	P*	P*	P*	P*
Arc length & area of sector	P*	P*	P*	P*
Small angle approximations	P*	P*	P*	P*
Graphs of sine, cos & tan	P	P	P	P
- transformations (incl. comb'ns)		P		
Exact values of sine, cos & tan - using degrees	P	P	P*(sic)	error?
- using radians	P*	P*	P*	P*
Reciprocal Trig. f'ns	P*	P*	P*	P*
- graphs	P*	P*	P*	P*
Inverse Trig. f'ns	P*	P*	P*	P*
- graphs	P*		P*	P*
$\tan\theta = \frac{\sin\theta}{\cos\theta}$	P	P	P	P
$\sin^2\theta + \cos^2\theta = 1$	P	P	P	P
$\tan^2\theta + 1 = \sec^2\theta$	P*	P*	P*	P*
$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$	P*	P*	P*	P*
Compound (incl. double) angle formulae	P*	P*	P*	P*
- geometrical proofs	P*	P*	P*	P*
$R\cos(\theta \pm \alpha)$ &	P*	P*	P*	P*

$\& R\sin(\theta \pm \alpha)$ forms					
Derivatives of inverse trig. fns				P*	CP*
t formulae: trig. identities & eq'n's					FP1
<b>Vectors</b>		P	P	P	P
Magnitude & direction		P	P	P	P
Convert between component & magnitude-direction forms	P	P	P	P	
Geometrical interpretation of addition of vectors, and multiplication by scalar	P	P	P	P	
Position vectors	P	P	P	P	
Distance between 2 points	P	P	P	P	
Scalar product (angle between 2 lines)	Y	PC	CP	P	CP
Eq'n of line (2D&3D)	Y	PC	CP*	P	CP
- direction cosines					FP1*
Eq'n of plane	Y	PC*	CP	P*	CP
Intersection of 2 lines	Y	PC	CP*	P	CP
Intersection of line & plane	Y	PC*	CP*	P*	CP
Angle between line & plane	Y	PC*	CP*	P*	CP
Angle between 2 planes	Y	PC*	CP*	P*	CP
Distance between point & line (or 2 parallel lines)	Y	PC*	CP*	P	CP
Distance between point & plane	Y	PC*	CP*	P*	CP

Distance between 2 skew lines	Y	PC*	CP*	P	CP
Vector product	Y	PC AP	CP*	P*	FP1
- eq'n of line		AP		P*	FP1*
- area of triangle		AP		P*	FP1*
Scalar triple product	Y	AP*			FP1