Ellipses Q3 [6 marks] (23/5/21)

Exam Boards

OCR : -

MEI: -

AQA: -

Edx: Further Pure 1 (Year 2)

Show that the area within the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab

[6 marks]

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[6 marks]

Solution

Area $= 4\int_0^a b\sqrt{1 - \frac{x^2}{a^2}}\, dx$ [1 mark]

Let $x = asin\theta$, so that $dx = acos\theta\, d\theta$ [1 mark]

Then Area $= 4b\int_0^{\frac{\pi}{2}} cos\theta . acos\theta\, d\theta$ [1 mark]

$= 2ab\int_0^{\frac{\pi}{2}} 1 + cos2\theta\, d\theta$ [1 mark]

$= 2ab[\theta + \frac{1}{2}sin2\theta]_0^{\frac{\pi}{2}}$ [1 mark]

$= 2ab\left(\frac{\pi}{2}\right) = \pi ab$ [1 mark]