Ellipses Q1 – Practice/E (23/5/21)

Show that the equation of the tangent to the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point (x_1, y_1) is $\frac{yy_1}{b^2} + \frac{xx_1}{a^2} = 1$

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Solution

Differentiating gives $\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$, so that $\frac{dy}{dx} = -\frac{xb^2}{ya^2}$

Then the tangent at (x_1, y_1) is $\frac{y-y_1}{x-x_1} = -\frac{x_1 b^2}{y_1 a^2}$,

or $yy_1a^2 - {y_1}^2a^2 = -xx_1b^2 + {x_1}^2b^2$

and hence $\frac{yy_1}{b^2} - \frac{{y_1}^2}{b^2} = -\frac{xx_1}{a^2} + \frac{{x_1}^2}{a^2}$

or $\frac{yy_1}{b^2} + \frac{xx_1}{a^2} = \frac{{x_1}^2}{a^2} + \frac{{y_1}^2}{b^2}$

As (x_1, y_1) lies on the ellipse, $\frac{{x_1}^2}{a^2} + \frac{{y_1}^2}{b^2} = 1$,

so we have $\frac{yy_1}{b^2} + \frac{xx_1}{a^2} = 1$ as the equation of the tangent.