

Differentiation Q4 – Practice/Y2/E (22/5/21)

Show that $\int \frac{1}{\sqrt{1+a^2x^2}} dx = \frac{1}{a} \ln \left| \sqrt{1+a^2x^2} + ax \right| + c$, by differentiation

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Solution

$$\frac{d}{dx} \left(\frac{1}{a} \ln \left| \sqrt{1+a^2x^2} + ax \right| \right) = \frac{1}{a} \cdot \frac{\frac{1}{2}(1+a^2x^2)^{-\frac{1}{2}}(2a^2x)+a}{\sqrt{1+a^2x^2}+ax}$$

Writing $A = \sqrt{1+a^2x^2}$, this gives:

$$\frac{A^{-1}ax+1}{A+ax} = \frac{1}{A} \cdot \frac{ax+A}{A+ax} = \frac{1}{A}, \text{ as required}$$