

**Differentiation - Exercises** (2 pages; 19/10/18)

(1) Find the derivative of  $\tan x$  using (a) the Quotient rule, and  
(b) the Product rule

(2) Find the turning points of  $y = (x^2 - 4x + 3)^2$

(3) If  $f(x) = x^2$ , what is  $f'(3x)$ ?

(4) Find  $\frac{d}{dx}(x^{\sin x})$

(5) Find  $\frac{d}{dx}(a^x)$

(6) (i) Give a definition for a point of inflexion.

(ii) Give an example of a turning point where  $f''(x) = 0$

(7) For a particular point on a curve:

S = stationary point

T = turning point

PI = point of inflexion

2D0 = 2nd derivative is zero

TG = turning point of gradient

Which of the following are true?

(a)  $T \Rightarrow S$

(b)  $S \Rightarrow T$

(c)  $PI \Rightarrow S$

(d)  $2D0 \Leftrightarrow PI$

(e)  $PI \Leftrightarrow TG$

(f)  $TG \Rightarrow 2D0$

(8) Referring to the abbreviations in (7), represent the events S, T, PI, 2D0 & TG of (ii) in a Venn diagram, showing where the following functions lie:

$$A: y = x \quad B: y = x^2 \quad C: y = x^3 \quad D: y = x^4 \quad E: y = \tan x \quad F: y = e^x$$

(ie depending on whether these functions exhibit any of the events)

Are there any regions of the Venn diagram that aren't satisfied by any functions?

(9) If  $f(x) = \sin x$ , express  $f^{(n)}(0)$  in terms of n, when n is odd (where  $f^{(n)}(x)$  denotes the nth derivative of  $f(x)$ )

(10) Show that  $\int \frac{1}{\sqrt{1+a^2x^2}} dx = \frac{1}{a} \ln \left| \sqrt{1+a^2x^2} + ax \right| + c$ , by differentiation