Differentiation - Exercises (2 pages; 18/2/20)

Key to difficulty:

* easier

** moderate

*** harder

(1*) Find the derivative of $tanx$ using (a) the Quotient rule, and (b) the Product rule

(2***) Find the turning points of $y = (x^2 - 4x + 3)^2$

(3**) If $f(x) = x^2$, what is $f'(3x)$?

(4***) Find $\frac{d}{dx}(x^{sinx})$

(5***) Find $\frac{d}{dx}(a^x)$

(6**) For a particular point on a curve:

S = stationary point

T = turning point

PI = point of inflexion

2D0 = 2nd derivative is zero

TG = turning point of gradient

Which of the following are true?

(a) $T \Rightarrow S$

(b) $S \Rightarrow T$

(c) $PI \Rightarrow S$

(d) $2D0 \Leftrightarrow PI$

(e) $PI \Leftrightarrow TG$

(f) $TG \Rightarrow 2D0$

(7**) Referring to the abbreviations in (6), represent the events S, T, PI, 2D0 & TG of (ii) in a Venn diagram, showing where the following functions lie:

$A: y = x$ $B: y = x^2$ $C: y = x^3$ $D: y = x^4$ $E: y = tanx$ $F: y = e^x$

(ie depending on whether these functions exhibit any of the events)

Are there any regions of the Venn diagram that aren't satisfied by any functions?

(8**) If $f(x) = sinx$, express $f^{(n)}(0)$ in terms of n, when n is odd (where $f^{(n)}(x)$ denotes the nth derivative of $f(x)$)

(9***) Show that $\int \frac{1}{\sqrt{1+a^2x^2}} dx = \frac{1}{a} ln \left| \sqrt{1 + a^2x^2} + ax \right| + c$, by differentiation