Curve Sketching - Exercises (2 pages; 18/2/20)

Key to difficulty:

* easier

** moderate

*** harder

(1***) Sketch the graph of $\sqrt{x^2 - 2x + 1}$ for $0 \leq x \leq 2$

(2***) (i) What possible shapes might a cubic have (ignoring its position relative to the axes)?

(ii) How many stationary points does the cubic function,

$f(x) = x^3 + x^2 - 2x + 3$ have?

(iii) What is the condition for there to be 2 stationary points for the general cubic $f(x) = ax^3 + bx^2 + cx + d$?

(iv) For $f(x) = ax^3 + bx^2 + cx + d$, find the x-coordinate of any turning points of the gradient.

If the cubic has turning points, how could they be used to find the point of inflexion?

(v) For $f(x) = ax^3 + bx^2 + cx + d$, find conditions for the shape of the curve to be each of the 3 possibilities shown in (i), by considering the gradient at the point of inflexion.

(3**) Sketch $y = |x - 2| + 1$

(4***)(i) Sketch the curve $y = \frac{4x^2+5x+7}{2x+3}$.

(ii) Without using calculus, find the coordinates of the stationary points (to 3sf).

(5***)(i) Find a series of transformations that can be applied to $y = \frac{1}{x}$ to produce $y = \frac{3x-2}{6x-1}$.

(ii) Hence or otherwise, sketch the curve $y = \frac{3x-2}{6x-1}$.