Cubic Graphs - Exercises (3 pages; 29/7/16)

(1) Point of Inflexion (or 'inflection')

This can be defined as a turning point of the gradient.

So $\frac{d}{dx}\left(\frac{dy}{dx}\right) = 0$ and $\frac{d^2}{dx^2}\left(\frac{dy}{dx}\right) \neq 0$

(sufficient but not necessary condition)

ie $\frac{d^2y}{dx^2} = 0$ & $\frac{d^3y}{dx^3} \neq 0$

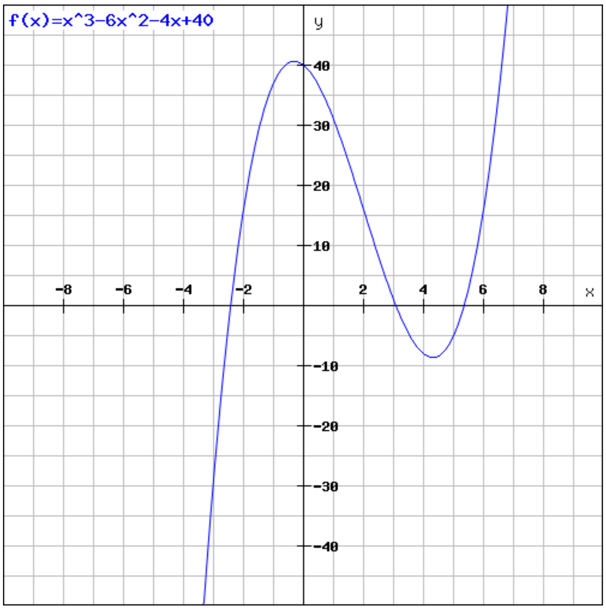

Fig. 1: $p(x) = x^3 - 6x^2 - 4x + 40$

Point of Inflexion at (2, 16)

(2) For $f(x) = ax^3 + bx^2 + cx + d$, what is the x-coordinate of the PoI?

(3) Give examples of cubic functions for which the PoI is at the Origin, and the gradient at the Origin is (a) 1 (b) -1. How do the shapes of the two graphs differ?

(4) Compare the x-coordinate of the PoI with those of the turning points (where they exist).

(5) Translate the function $q_0(x) = 2x^3 + x$ by $\begin{pmatrix} 3 \\ 20 \end{pmatrix}$ and confirm the x-coordinate of the PoI of the translated function.

(6) Find the function $p_0(x)$ that results from translating $p(x) = x^3 - 6x^2 - 4x + 40$, so that its PoI is at the Origin. Sketch $p_0(x)$

(7) Consider $g(x) = x^3 + bx^2 + cx + d$

Translate the graph of $g(x)$ so that its PoI is at the Origin (to give $g_0(x)$).

(8) Consider the effect of reflecting $g_0(x) = x^3 + c_0x$ in the x-axis and then in the y-axis. What does this reveal?

(9) Show that the PoI of $y = a(x-p)(x-q)(x-r)$ is at $x = \frac{1}{3}(p+q+r)$

(10) By considering the gradient at the Origin, sketch the possible shapes of $g_0(x) = x^3 + (c - \frac{b^2}{3})x$

(11) Consider $f(x) = ax^3 + bx^2 + cx + d$

Find $f_0(x)$, the translated function with its PoI at the Origin.

(12) What condition must apply to b & c for the function $f(x) = ax^3 + bx^2 + cx + d$ to have two turning points?