Critical Path Analysis - Part 2 (13 pages; 10/11/18)

Resourcing and Scheduling

(1) Introduction

(1.1) Important considerations for a project are usually:

(a) minimising the completion time

(b) minimising the number of workers

(c) minimising costs

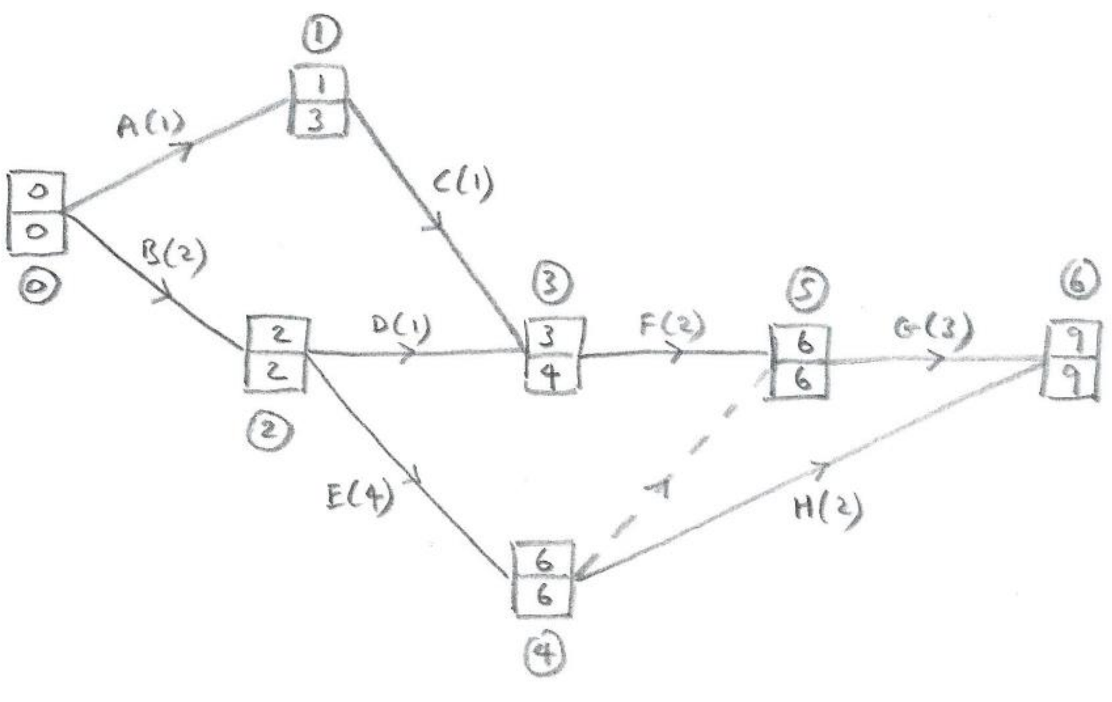
(1.2) There are three charts that can be created to help with resourcing decisions:

Gantt (or Cascade) chart

Resource histogram

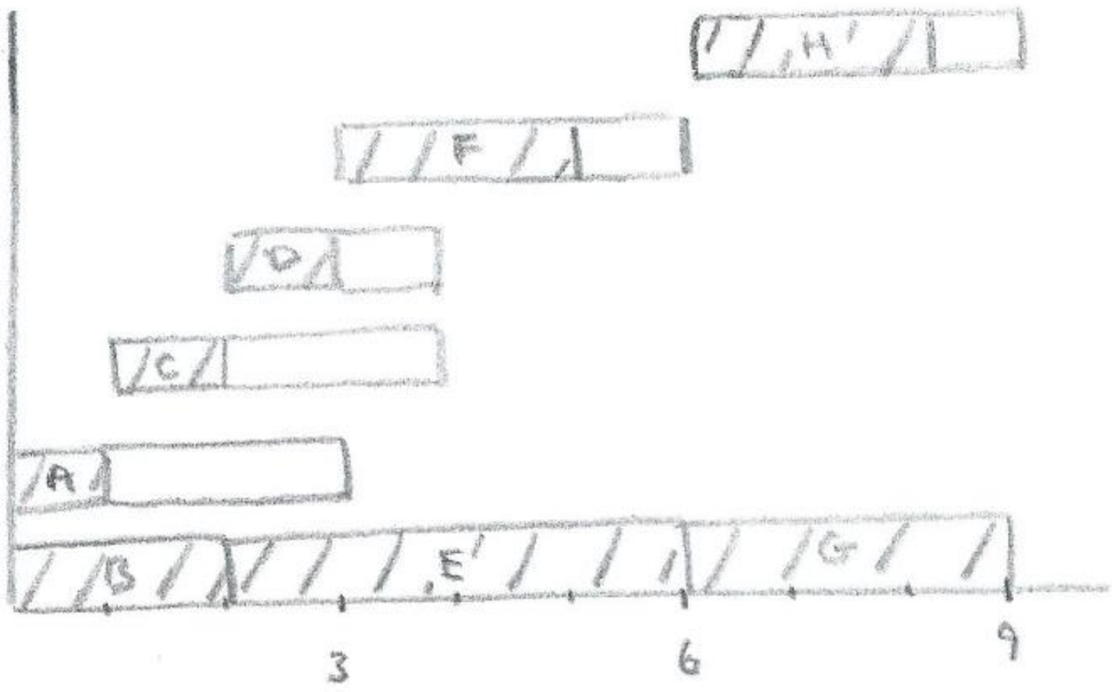
Scheduling diagram

These will be illustrated for the following activity network (using activity-on-arc):



(The critical path is B, E, G.)

(2) Gantt (or Cascade) chart



Notes

(i) The critical activities may appear in one row (as in the diagram), but are sometimes given separate rows. There may be more than one critical path.

(ii) Each of the non-critical activities is assumed to start at the earliest possible time; eg for F, the earliest event time for node 3 (which is also referred to as the earliest start time for F).

(iii) The shaded sections of the bars are the durations of the activities, with the floats being unshaded. (Sometimes there is no shading, and the float is indicated by a dotted border.)

(iv) The number of workers required for each activity is sometimes shown on the bar.

(v) The activity precedences are not recorded in the diagram (eg the fact that C has to take place before F).

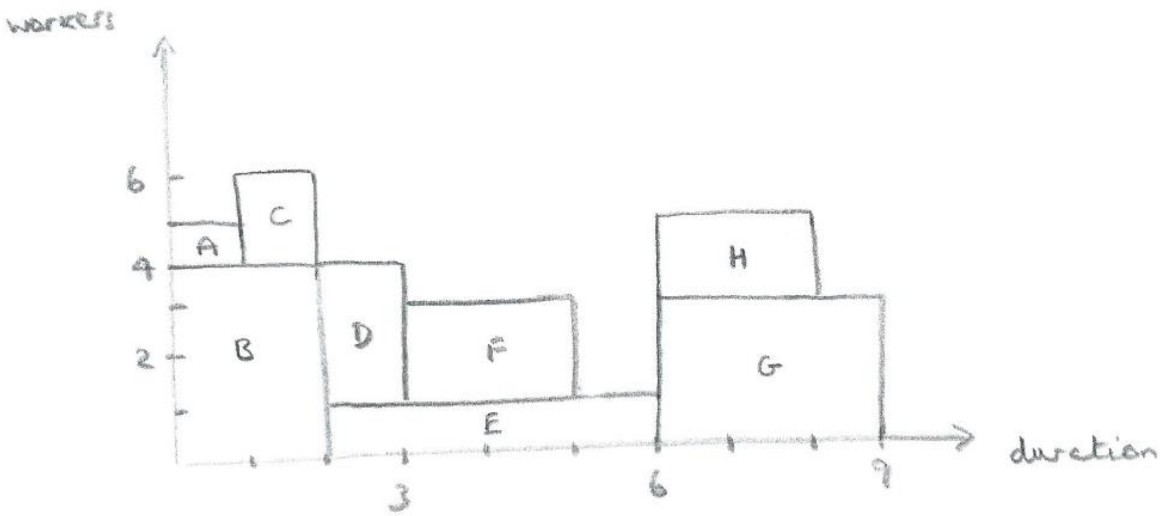
A Gantt chart can be used to establish which activities definitely have to be taking place at a given point in time (ie activities that can't be shuffled away from a vertical line at that point).

(3) Resource Histogram

The numbers of workers needed for the activities in the above example are shown in the table below.

Activity	A	B	C	D	E	F	G	H
Workers	1	4	2	3	1	2	3	2

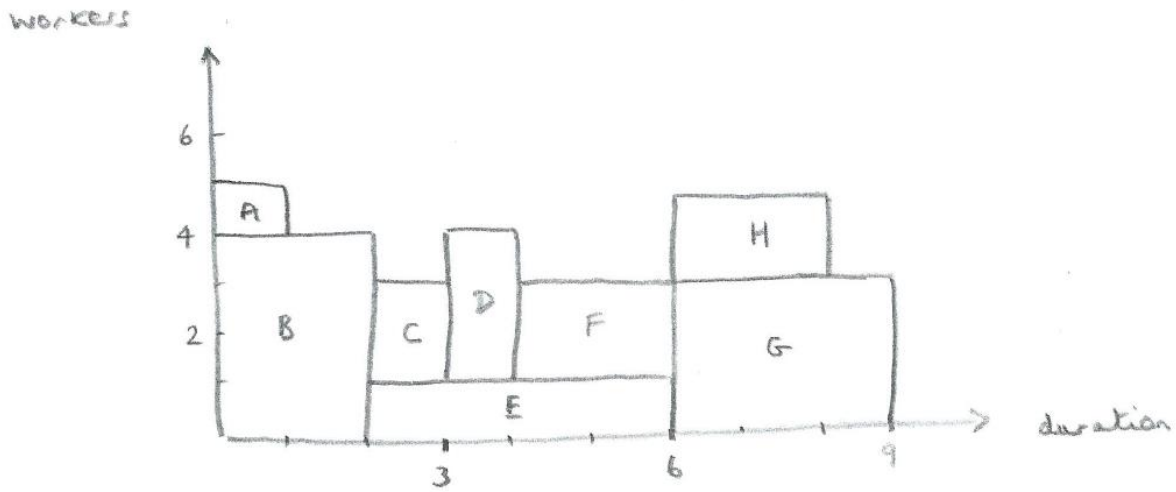
This enables the Resource histogram to be constructed, as below.



As can be seen, the width of each activity is the same as in the Gantt chart.

(4) Resource Levelling

(4.1) Sometimes it is possible to shuffle activities along (within their boxes in the Gantt chart), in such a way as to reduce the total number of workers required at any one time (without increasing the completion time). This then leads to a revised Resource histogram - see below for the example being considered. Here we see that 5 workers are needed.



Because the activity precedences are not recorded in the Gantt chart, we have to be careful that the revised Resource histogram doesn't infringe those precedences.

In a simple case, where only one worker is needed for each activity, the activities in the Gantt chart can often be shuffled visually. For example, if it is possible to shuffle the activities so that no vertical line crosses more than 3 activities, then 3 workers will be sufficient.

(4.2) A lower bound for the smallest possible number of workers needed can be found by considering the ideal situation in which the activities are shuffled into a rectangle, the base of which is the minimum completion time.

For the current example, the area of the rectangle is the number of man-days needed:

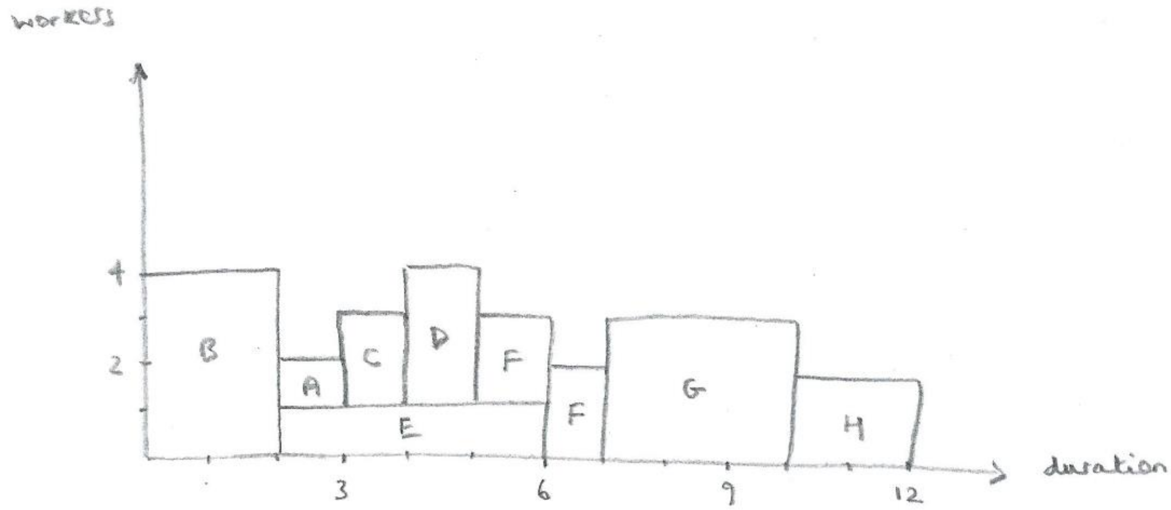
$1 + 8 + 2 + 3 + 4 + 4 + 9 + 4 = 35$,

where the number of man-days for B, for example, is $2 \times 4 = 8$

Thus the lower bound is $\frac{35}{9} = 3.89$ (3sf); ie 4 workers, as a whole number is required.

Also, 4 workers are required for activity B, which in general places a further constraint on the lower bound.

(4.3) It may be the case that only 4 workers are available (in this example), and the completion time has to be extended. The Resource histogram can then be adapted to achieve this (see diagram below). Note that some activities will move outside their boxes in the Gantt chart, and that the precedences will need to be taken into account.



Note that activity F is in two parts (this avoids having an empty space below the 2nd half of F).

(5) Scheduling Diagram

[The following is my understanding of the standard procedures to be followed. However there is rarely universal agreement about anything in Decision Maths. Any grey areas will probably either be clarified in an exam question, or else avoided.]

(5.1) Each worker is allocated a row containing his/her activities.

The simplest situation is where only one worker is required for each activity, and where the aim is to find the minimum number of workers required, in order to complete the project in the minimum completion time.

There is a standard procedure to be followed:

(i) Allocate the critical activities to the 1st worker (who will have no time available for further activities, as the project is to be completed in the minimum completion time).

(ii) From the Gantt chart, list the remaining activities in order of increasing earliest start time.

(iii) Allocate the first of these activities to the 2nd worker. If two activities have the same earliest start time, choose one that has the lowest latest finish time (for maximum flexibility).

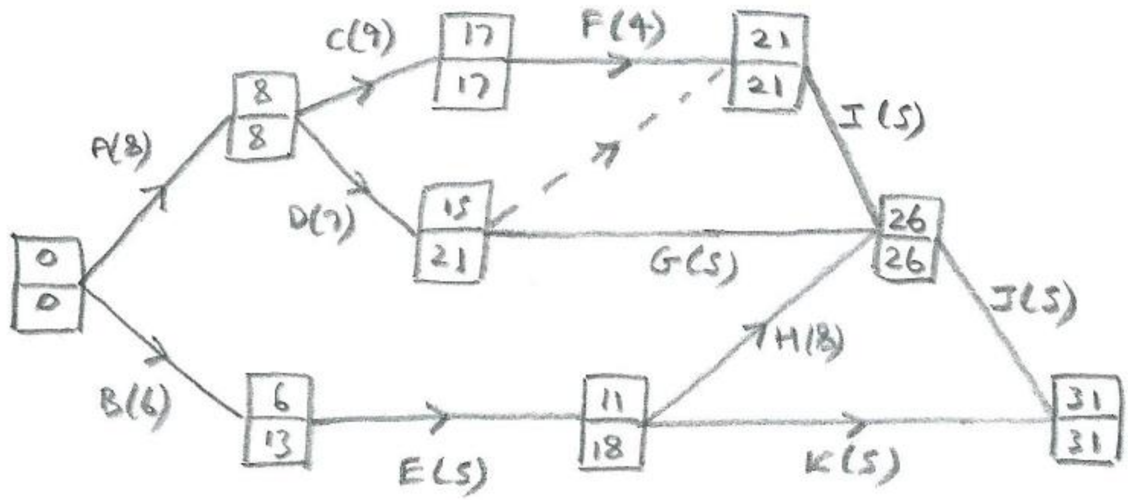
(iv) Allocate subsequent activities (in the same way) to the 2nd worker, if possible - by delaying the start date, if necessary. (But ensure that the activity precedences are not contravened.)

(v) If this is not possible, then allocate the activity to the 3rd worker.

(vi) Allocate the remaining activities to the first available worker, adding additional workers where necessary.

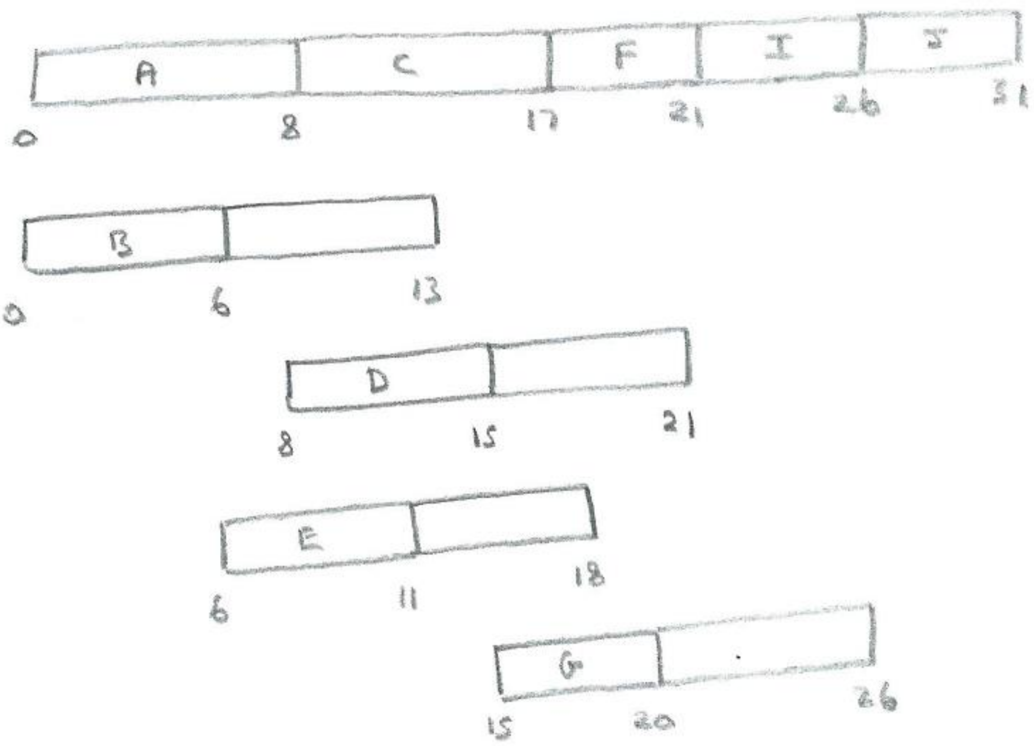
Example

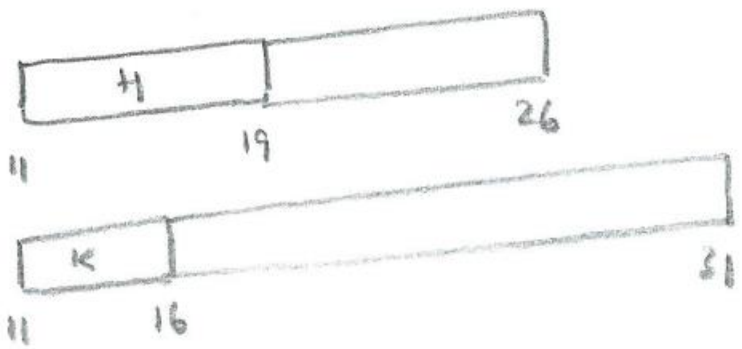
Apply the standard scheduling procedure to the following network:



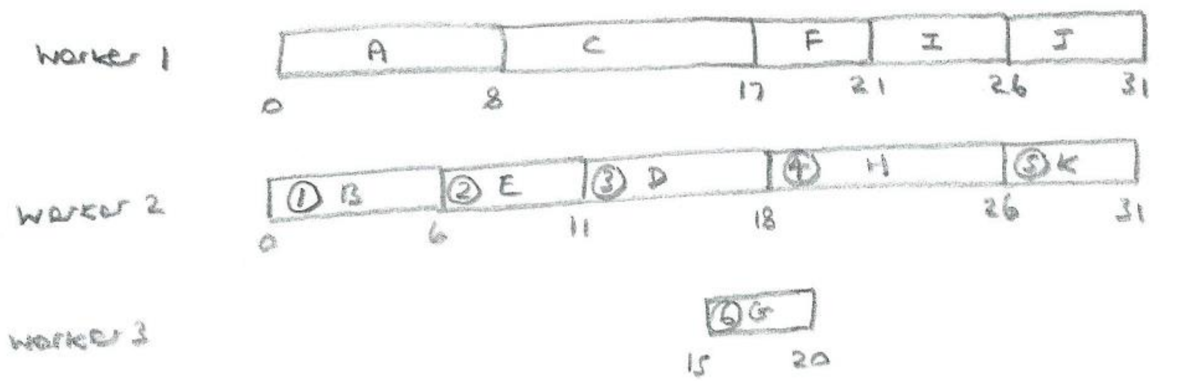
Solution

Gantt chart:





Scheduling diagram:



(5.2) Alternatively, the aim may be to schedule the workers when a limited number are available (say n), and the completion time therefore has to go beyond the minimum possible.

Assuming again that only one worker is required for each activity, the standard procedure is now as follows:

(i) The critical activities are treated in the same way as the other activities (and are not allocated to worker 1).

(ii) From the Gantt chart, list the activities in order of increasing earliest start time.

(iii) Allocate activities to the next available worker. If two activities have the same earliest start time, choose one that has the lowest latest finish time.

(iv) If there is more than one activity with the same latest finish time, then a number of possibilities will need to be explored, to see which gives the shortest completion time.

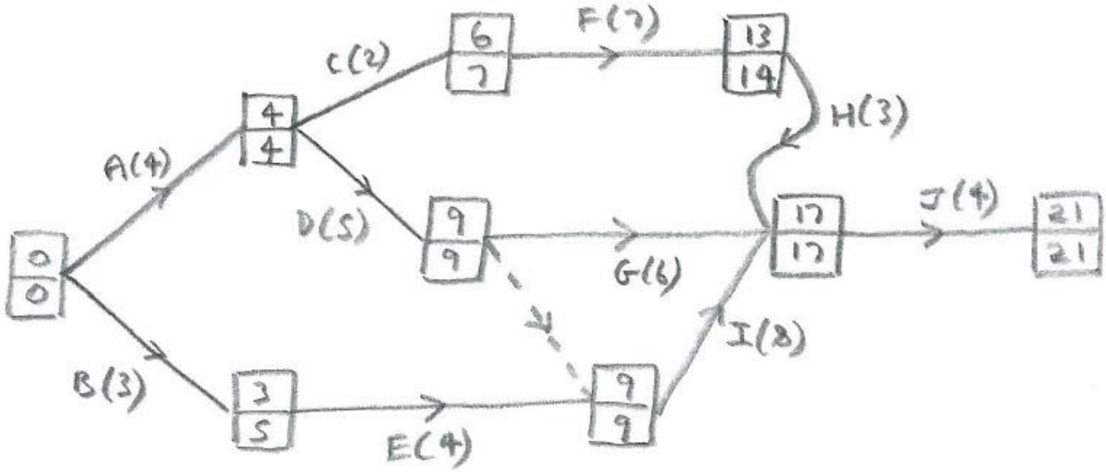
(v) Ensure that the activity precedences are not contravened.

However, as will be seen in the next example, this doesn't always give the best solution.

[Note that the procedure involves the earliest start and latest finish times, even though these are only strictly applicable when there is no constraint on the number of workers. This is glossed over in textbooks.]

Example

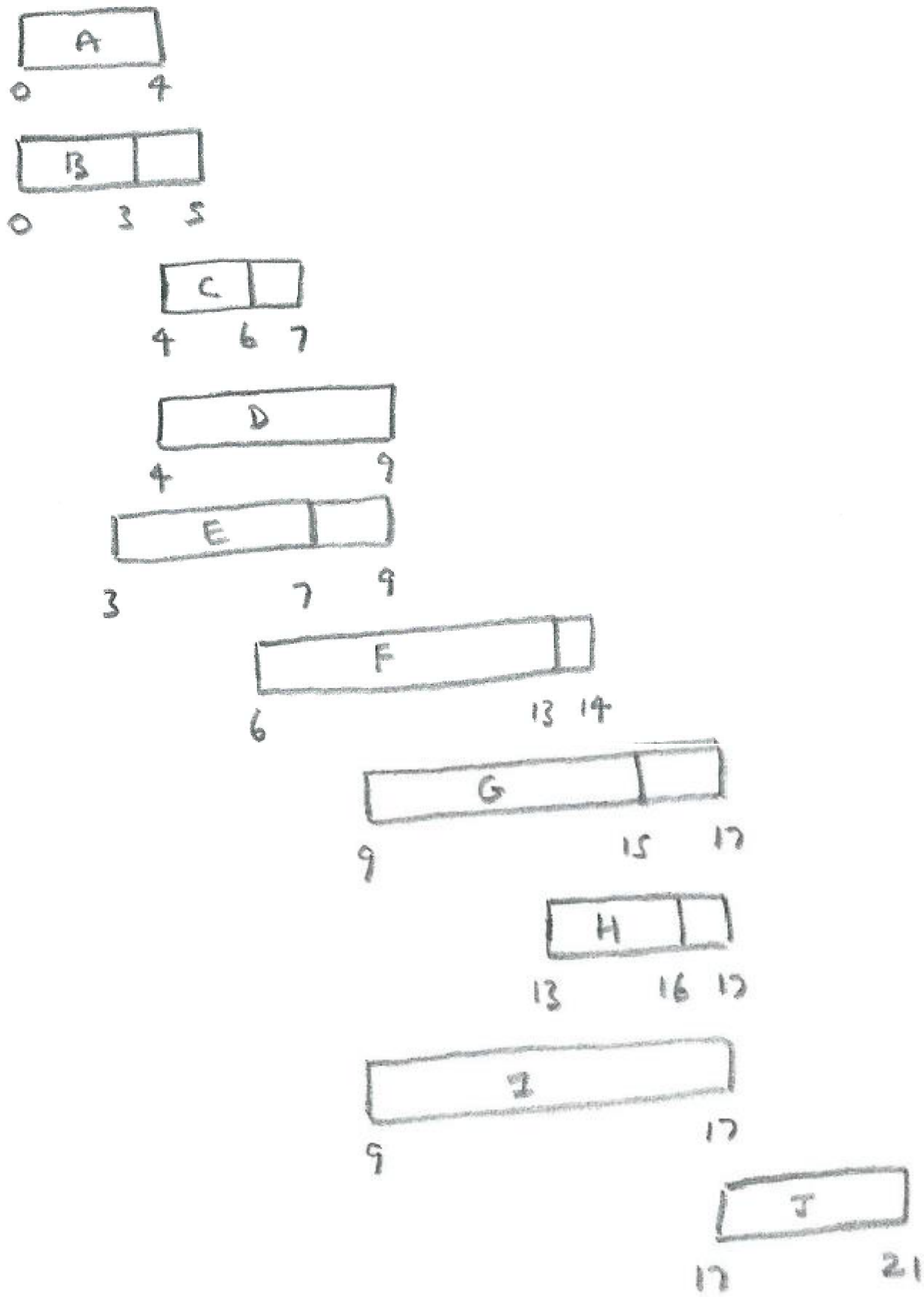
(i) Apply the standard scheduling procedure to the following network, when only two workers are available.



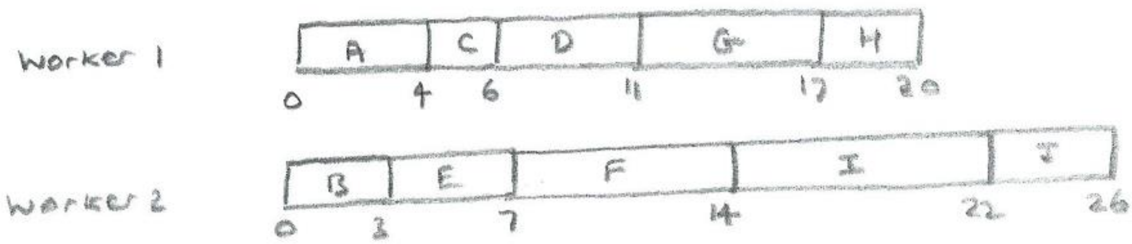
(ii) Find an improved schedule, such that the project can be completed in 25 days.

Solution

(i) Gantt chart:

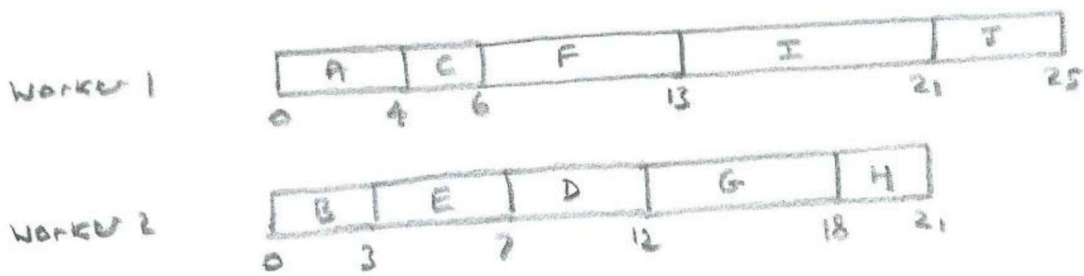


Scheduling diagram:

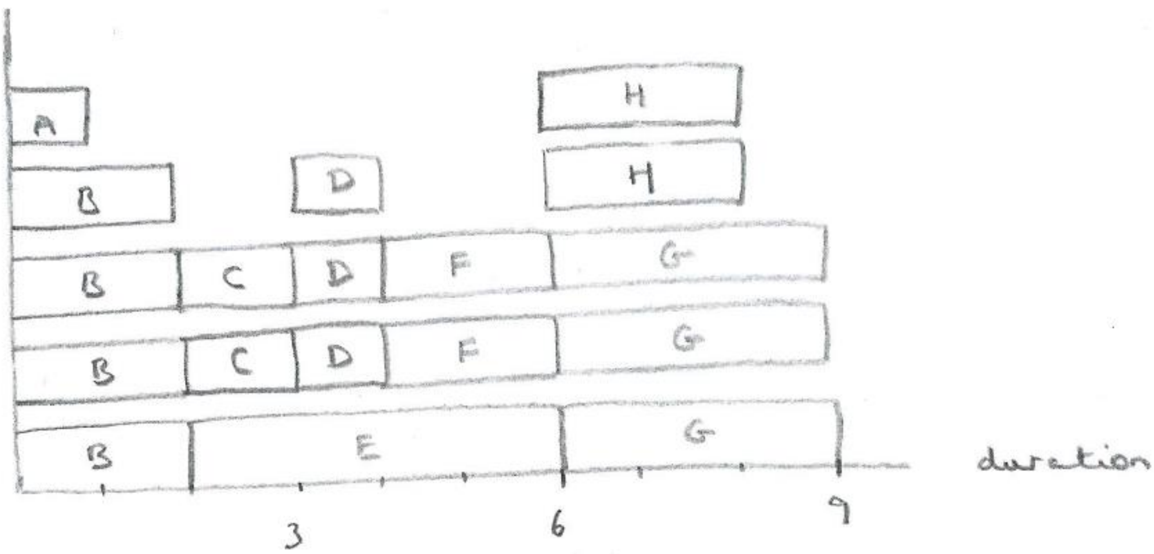


(this involves choosing G in preference to I - they both have the same earliest start and latest finish times)

(ii) Juggling around the activities in (i):



(5.3) The example considered in (4.1), when resource levelling was carried out, involved more than one worker for each activity. For that example, a scheduling diagram for 5 workers (with the project being completed in the minimum completion time) can be derived directly from the Resource histogram (after levelling), as shown below.



(6) Crashing a network

If it is especially important to minimise the completion time, then the following measures might be taken:

(i) increase the number of workers; eg if an activity takes 4 days when carried out by 1 worker, it may be the case that it can be completed in 2 days by 2 workers

(ii) reduce the duration of an activity by some other expenditure (eg hiring a machine)

These measures are referred to as 'crashing a network'. In both cases, a cost is associated with reducing the duration of an activity by a certain amount.

Note that it is the critical activities that need to be shortened first. Beyond a certain point though, other activities may become critical. The activity network may need to be re-drawn.

Example

The table below shows the costs associated with the original durations for each activity, for the example considered earlier, in (1). It is possible to reduce some of the durations of the activities, at an extra cost, as shown in the table. If the duration is reduced only partially, then the new cost is calculated on a pro-rata basis; eg for activity E, the cost for 3 days would be £750.

Activity	A	B	C	D	E	F	G	H
Original duration (days)	1	2	1	1	4	2	3	2
Cost (£)	100	300	200	100	500	400	300	300
New duration (days)	1	1	1	1	2	1	1	1
Cost (£)	100	500	200	100	1000	600	900	500

Assuming that there are no constraints on the number of workers available, which activities would you recommend be speeded up, if the required reduction in the total time were:

(a) 1 day (b) 2 days (c) 3 days?

And (d) What is the biggest possible reduction, and what is the extra cost associated with this?

Solution

The options for achieving reductions in the durations of activities are:

(1) Reduce B by 1 (extra cost: £200)

(2) Reduce E by 1 (extra cost: £250)

(3) Reduce E by 2 & F by 1 (extra cost: 500+200=£700)

(4) Reduce G by 1 (extra cost: £300)

(5) Reduce G by 2 & H by 1 (extra cost: 600+200=£800)

(a) recommend (1)

(b) recommend (1) & (2)

(c) recommend (1) & (3)

(d) (1), (3) & (5) (reduction of 5, at extra cost of 200+700+800=£1700)

A systematic way of finding the shortest completion time is to create a new activity network, based on all the minimum durations, and then increase the durations of any non-critical activities (ie so that the floats are used up).