

Counting Q5 [Problem/H] (9/6/21)

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Solution

First of all, note that ABCDEF and BCDEFA will (usually) be considered to be the same seating arrangement, as the table is circular.

Then, without loss of generality, we can start with B.

Method 1

Consider 3 separate cases: BXEXXX, BXXEEX & BXXXEX (these are the permissible cases). [Note that E can't be at the end, as then they would be next to B at the circular table.]

In each case, there are $4!$ possible arrangements.

Hence there are $3 \times 4! = 72$ possible arrangements overall.

Method 2

Starting with B, there are $5!$ ways of filling the remaining 5 places (including non-permissible arrangements).

From these, deduct the non-permissible arrangements, which are of the form BEXXXX or BXXXXE

There are $2 \times 4!$ of these, giving a final answer of

$$5! - 2 \times 4! = 120 - 48 = 72$$