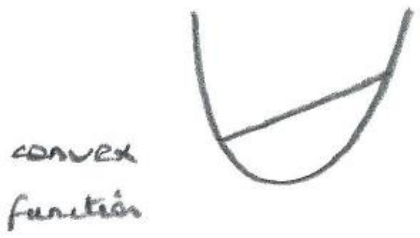
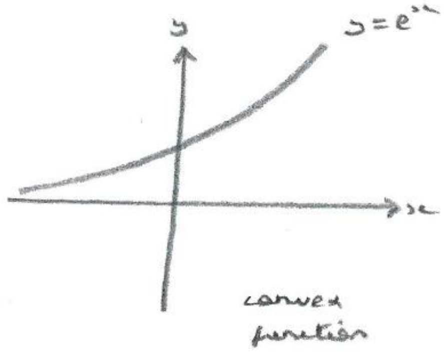
Convex & Concave Functions (2 pages; 2/6/23)

(1) Examples of a concave function are $y = x^2$ and $y = e^x$ (think of conve^x!).

For a convex function, the line connecting any two points on the curve lies entirely above the curve.

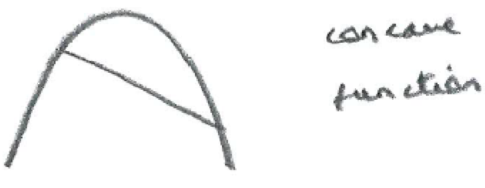


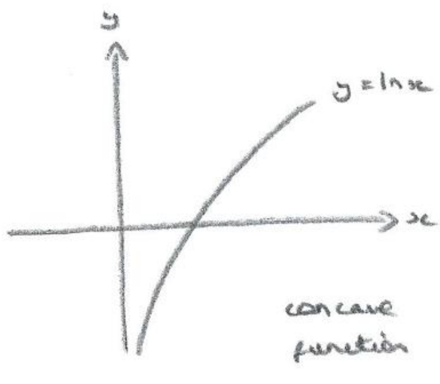


This can be expressed mathematically as:

$f(\lambda a + (1 - \lambda)b) < \lambda f(a) + (1 - \lambda)f(b)$ for any a & b in the domain of the function $y = f(x)$; ie the curve lies below the line joining a & b

(2) Examples of a concave function are $y = -x^2$ and $y = lnx$. For a concave function, the line connecting any two points on the curve lies entirely below the curve.





Mathematically,

$f(\lambda a + (1-\lambda)b) > \lambda f(a) + (1-\lambda)f(b)$ for any a & b in the domain of the function $y = f(x)$; ie the curve lies above the line joining a & b

(3) Alternatively, a function is convex when $\frac{d^2y}{dx^2} > 0$, and concave when $\frac{d^2y}{dx^2} < 0$.

A point of inflexion occurs when $\frac{d^2y}{dx^2}$ changes sign. This is when a function changes from convex to concave (or the reverse).