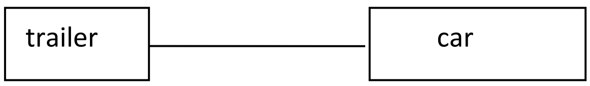
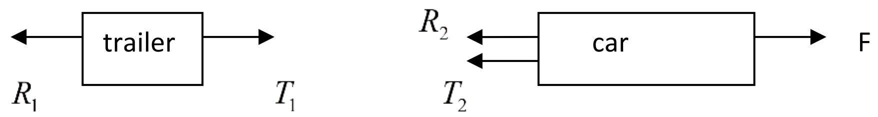
Connected Particles (7 pages; 24/10/18)

(1) Car towing trailer, by means of a towbar

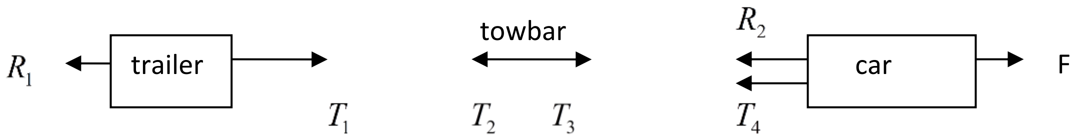


Force Diagrams (method 1)



Here the towbar is being ignored. The car is exerting a force of T_1 (via the towbar) on the trailer, and the trailer is exerting a force of T_2 on the car. By Newton's 3rd law, $T_1 = T_2$. R_1 and R_2 are the resisting forces on the trailer and the car, respectively (due to friction and air resistance).

Force Diagrams (method 2)



Here the towbar is considered as a separate object. The car exerts a force of T_3 on the towbar, which exerts a force of T_4 on the car, and again, by Newton's 3rd Law, $T_3 = T_4$. Similarly, for the trailer, $T_1 = T_2$.

The towbar is thus being pulled from each end, and is therefore under tension (hence the symbol T).

Note: If the tension in the towbar is not required, then a single force diagram can be drawn for the combined car & trailer object.

In the general case, where the vehicles are accelerating and the towbar's mass is not negligible, $T_3 \neq T_2$, since the towbar is also accelerating and requires a force to do so ($T_3 - T_2$).

If the towbar is assumed to have negligible mass (as is usually the case), then no force is required to accelerate it, and so $T_3 = T_2$

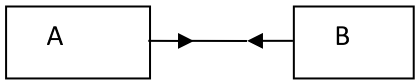
Also, if the vehicles (and the towbar) are not accelerating, then $T_3 = T_2$ again.

Then, provided $T_3 = T_2$, $T_1 = T_2 = T_3 = T_4$.

[The assumption of negligible mass is in fact necessary for method 1 to be valid – ie for the towbar to be ignored.]

In some cases of deceleration, T_1 will be negative (since the acceleration of the trailer is T_1 - R_1 and, if R_1 is small, T_1 may have to be negative in order to obtain the necessary braking); in other words, the towbar will be in compression (sometimes called 'thrust').

Although separate force diagrams make things clearer, the following convention is usually followed for objects joined by a rod or string etc.



Here the arrows indicate the forces on the objects A and B. By Newton's 3rd Law, A and B both pull on the connecting rod, which is therefore under tension.

(2) Car and caravan - Example

A car of mass 1000 kg is pulling a caravan of mass 1500 kg. The car and caravan are connected by a light towbar. The total resistive forces on the car and caravan are 150 N and 200 N, respectively (throughout the motion).

The car is travelling at $20ms^{-1}$ when it brakes, so as to decelerate at $0.5ms^{-2}$.

Find

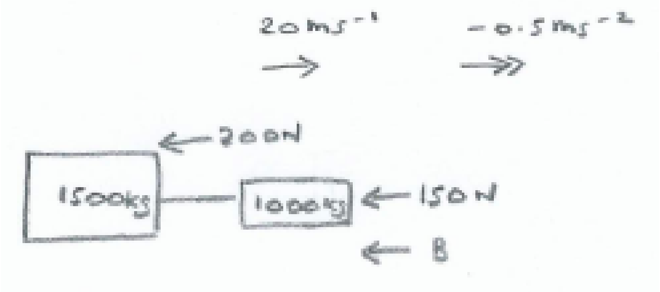
(i) the braking force

(ii) the tension or compression in the towbar

(iii) the distance travelled by the car and caravan before coming to rest

(iv) the time taken to come to rest

Solution



(i) Considering the car, caravan & towbar as a single object,

$N2L \Rightarrow B + 200 + 150 = (1500 + 1000)(0.5)$,

where B is the braking force

Hence $B = 900\ N$

[the tension is ignored, as it is an internal force (internal forces come in equal and opposite pairs, by N3L); although the braking force is generated by the car's engine, it can be treated as an external force; the resistive forces include both air resistance (or wind) and any friction causing the car to slow down]

(ii) Considering the caravan,

$N2L \Rightarrow T - 200 = (1500)(-0.5)$,

where T is the tension in the towbar

Hence $T = -550\ N$

ie a compression of 550 N

[Check: Considering the car,

$N2L \Rightarrow -900 - 150 - T = (1000)(-0.5) \Rightarrow T = -550$]

(iii) $v^2 = u^2 + 2as \Rightarrow$

$0 = 20^2 + 2(-0.5)s$

Hence distance travelled, $s = 400m$

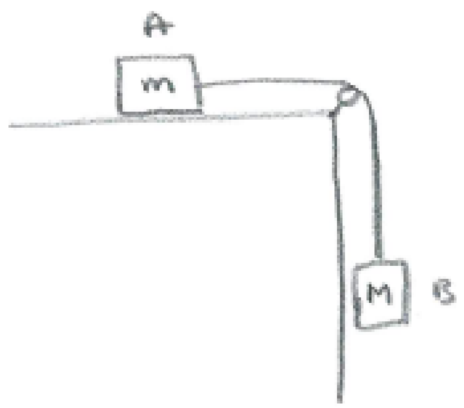
(iv) $v = u + at \Rightarrow$

$0 = 20 + (-0.5)t$

Hence time taken to come to rest, $t = 40s$

[Check: $s = ut + \frac{1}{2}at^2 = (20)(40) + \frac{1}{2}(-0.5)(1600) = 400$]

(3) Pulley Example



Referring to the diagram, where blocks A and B have masses m and M, respectively, and the coefficient of friction between block A and the surface is μ. (The connecting rope can be assumed to be light and inextensible, and the pulley is smooth.)

(i) What condition must apply to μ for the blocks to accelerate?

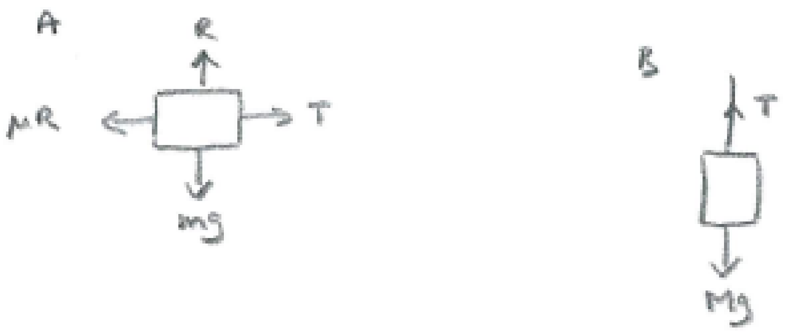
(ii) Hence, if block A has mass 1 kg and is on the point of slipping, find μ in terms of M.

(iii) If $\mu = 0$, what is the acceleration of the blocks (in terms of m, M and g)?

(iv) If block A has mass 1 kg and $\mu = 0.5$, what mass must block B have in order for its acceleration to be 0.5g?

Solution

(i) Creating separate force diagrams for the two blocks,



If block A is either slipping, or on the point of slipping,

$N2L \Rightarrow$

block A: $T - \mu R = T - \mu mg = ma \quad (1)$

block B: $Mg - T = Ma \quad (2)$

where a is the acceleration of the blocks

Adding (1) & (2) gives $g(M - \mu m) = (m + M)a$,

so that $a = \frac{g(M-\mu m)}{M+m} \quad (3)$

Thus, for the blocks to accelerate, $M - \mu m > 0$,

and $\mu < \frac{M}{m}$ (4)

(ii) From (3), $a = 0 \Rightarrow M - \mu m = 0$

so that $\mu = \frac{M}{m} = M$

(iii) From (3), $a = \frac{M}{M+m}\, g$

(iv) From (3), $0.5g = \frac{g(M-0.5)}{M+1}$

$\Rightarrow 0.5M + 0.5 = M - 0.5$

$\Rightarrow 1 = 0.5M \Rightarrow M = 2$

ie block M must have mass 2 kg