Conics - Exercises (2 pages; 16/1/17)

(1) Show that the equation of the tangent to the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(acosht, bsinht)$ is

$yasinht = xbcosht - ab$

(2) Given that the tangent in (1) meets the asymptotes of the hyperbola at the points P & Q, show that the mid-point of P & Q is $(acosht, bsinht)$.

(3) In the case where $b = a$, find the area of the triangle OPQ (where P & Q are as in (2), and O is the Origin).

(4) Given the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and circle $x^2 + y^2 = a^2$, let l_1 be the tangent to the ellipse at the point $(acos\theta, bsin\theta)$ and l_2 be the tangent to the circle at the point $(acos\theta, asin\theta)$. Find the locus of the point of intersection of l_1 & l_2, as θ varies.

(5) The chord PQ, where P and Q are points on the rectangular hyperbola $xy = c^2$, has gradient 1. Show that the locus of the point of intersection of the tangents from P and Q is the line

$y = -x$. [Edx FP3 textbook, Ex. 2G, Q9]

(6) Using the parametric equations of a parabola ($x = at^2$, $y = 2at$), show that the midpoints of chords of a parabola that have the same direction lie on a straight line parallel to the x-axis.

[A chord of a parabola joins two points on the parabola.]