Complex Numbers Exercises - Part 1 (3 pages; 13/2/16)

(1) Find $(2+5i) \div (1+3i)$ by two methods

(2) Solve the equation $(2+i)z+3=0$ by two methods

(3) Solve the equation $z^2-2z+2=0$

(a) by completing the square
(b) by equating real & imaginary parts

(4) Represent the following on the Argand diagram:

(i) $|z-i|>|z+1|$

(ii) $|z-i|=2|z+1|$

(5) $1+3i$ is a root of the equation $z^3+pz+q=0$ (where p & q are real). Find the other roots, and the values of p & q

(6) Find the square roots of $3-4i$

(7) Let $z=\frac{a+i}{1+ai}$. If $arg z=-\frac{\pi}{4}$, find the possible values of a

(8) For each of the following numbers, say whether they are imaginary or complex (or both):

(i) 1 (ii) i (iii) 0 (iv) $1+i$

(9) Are these statements true or false? (Give an explanation, or a counter example, as appropriate.)

(i) All imaginary numbers are complex numbers.

(ii) All complex numbers are imaginary numbers.

(iii) All real numbers are complex numbers.

(iv) Zero is an imaginary number.

(v) The imaginary part of a complex number is an imaginary number.

(vi) All complex numbers are either real numbers or imaginary numbers.

(vii) Two imaginary numbers added together can sometimes give a real number.

(viii) If two complex numbers multiply to give a real number, then they must be conjugates of each other.

(ix) The square root of a non-real complex number is never real.

(10) How are the complex numbers z and zi related?

(11) Find the solutions of $z^2 = i$ by

(a) setting $z = a + bi$ and equating real and imaginary parts

(b) using de Moivre's theorem

(12) Simplify $e^{i\pi} + 1$

(13) How are the complex numbers z and $\frac{1}{z}$ related to each other?

(14) Find $(1+i)^{10}$ by considering rotations and magnifications in the Argand diagram

(15) Show that, if ω is an nth root of unity, then ω^r is also (where n & r are positive integers).