

Complex Numbers – Q8 (22/5/21)

Exam Boards

OCR : Pure Core (Year 2)

MEI: Core Pure (Year 2)

AQA: Pure (Year 2)

Edx: Core Pure (Year 2)

Find the cube roots of -8 in cartesian form [8 marks]

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Solution

$$z^3 = 8(\cos\pi + i\sin\pi) \text{ [1 mark]}$$

$$z_1 = 2\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right) \text{ [1 mark]}$$

$$z_2 = 2\left(\cos\left(\frac{\pi}{3} + \frac{2\pi}{3}\right) + i\sin\left(\frac{\pi}{3} + \frac{2\pi}{3}\right)\right) = 2(\cos\pi + i\sin\pi)$$

[1 mark]

$$z_3 = 2\left(\cos\left(\frac{\pi}{3} + \frac{4\pi}{3}\right) + i\sin\left(\frac{\pi}{3} + \frac{4\pi}{3}\right)\right) = 2\left(\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)\right)$$

[1 mark]

$$= 2\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right) \text{ [1 mark]}$$

$$\text{ie } z_1 = 2\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 1 + \sqrt{3}i \text{ [1 mark]}$$

$$z_2 = -2 \text{ [1 mark]}$$

$$z_3 = 2\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 1 - \sqrt{3}i \text{ [} z^3 + 8 = 0 \Rightarrow z_1^* \text{ is root] [1 mark]}$$