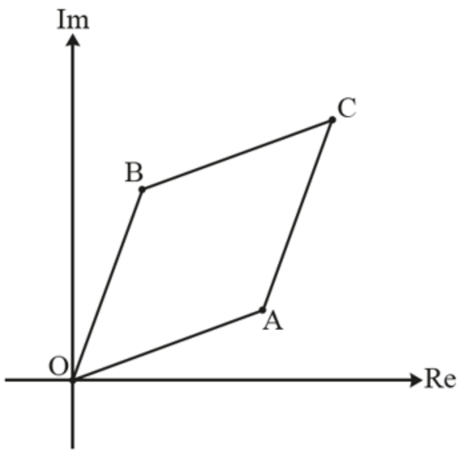
Complex Numbers Q25 – Problem/H (16/6/23)

Referring to the diagram, use complex numbers to prove that the diagonal OC of the rhombus OACB bisects the angle OAB.



Solution

Let z & w be the complex numbers represented by the points A & B. Write $z+w=re^{i\theta}+re^{i(\theta+\alpha)}$, where $\alpha=\angle AOB$

[aiming to show that arg $(z+w)$ will be $\theta+\frac{\alpha}{2}$]

Then $z+w=re^{i\left(\theta+\frac{\alpha}{2}\right)}(e^{-i\frac{\alpha}{2}}+e^{i\frac{\alpha}{2}})$

$=re^{i\left(\theta+\frac{\alpha}{2}\right)}.\ 2\cos(\frac{\alpha}{2}),$

and hence $\arg(z+w)=\theta+\frac{\alpha}{2}=\frac{1}{2}(\theta+[\theta+\alpha])$

$=\frac{1}{2}(argz+argw)$

Then, as C represents $z+w$, OC bisects the angle OAB.