Complex Numbers Q24 – Problem/H (15/6/23)

Use complex numbers to show that $sin\left(\frac{5\pi}{12}\right) = \frac{\sqrt{2}+\sqrt{6}}{4}$

Solution

$\frac{5\pi}{12} = \frac{\pi}{4} + \frac{\pi}{6}$

Let $z_1 = \cos\left(\frac{\pi}{4}\right) + isin(\frac{\pi}{4})$ and $z_2 = \cos\left(\frac{\pi}{6}\right) + isin(\frac{\pi}{6})$

Then $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2) = \frac{\pi}{4} + \frac{\pi}{6} = \frac{5\pi}{12}$,

and hence $sin\left(\frac{5\pi}{12}\right) = \frac{Im(z_1 z_2)}{|z_1 z_2|} = \frac{Im(z_1 z_2)}{|z_1||z_2|} = Im(z_1 z_2)$

Then, as $z_1 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$ and $z_2 = \frac{\sqrt{3}}{2} + \frac{1}{2}i$,

$Im(z_1 z_2) = \frac{1}{\sqrt{2}} . \frac{1}{2} + \frac{1}{\sqrt{2}} . \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{4}$, as required.