Complex Numbers Q23 – Practice/M (29/5/23)

Find the square roots of $-5 - 12i$

Solution

Let $(a + bi)^2 = -5 - 12i$

Then, equating Re. & Im parts:

$a^2 - b^2 = -5$ & $2ab = -12$

so that $a^2 - (\frac{-12}{2a})^2 = -5$

$\Rightarrow a^4 - 36 = -5a^2$

Writing $c = a^2, c^2 + 5c - 36 = 0$

$\Rightarrow (c + 9)(c - 4) = 0$

$\Rightarrow a^2 = 4$ (reject $a^2 = -9$, as negative)

$\Rightarrow a = \pm 2$

$a = 2 \Rightarrow b = \frac{-12}{2a} = -3$

and $a = -2 \Rightarrow b = 3$

So the square roots are $2 - 3i$ and $-2 + 3i$ [ie $\pm(2 - 3i\ )$]