Complex Numbers Q22 – Practice/M (29/5/23)

Points representing the 3 roots of the equation

$z^3 + z^2 - 7z - 15 = 0$ are plotted on an Argand diagram.

Given that one of the roots is an integer, find the area of the triangle that has these 3 points as its vertices.

Solution

Let $f(z) = z^3 + z^2 - 7z - 15$

If $f(z)$ is to factorise, then we need only consider factors of 15 when applying the Factor theorem.

$f(1) = 1 + 1 - 7 - 15 = -20$

$f(-1) = -1 + 1 + 7 - 15 = -8$

$f(3) = 27 + 9 - 21 - 15 = 0$

Thus $z - 3$ is a factor,

and we can write $z^3 + z^2 - 7z - 15 = (z - 3)(z^2 + 4z + 5)$

The roots are therefore 3 & $\frac{-4 \pm \sqrt{16-20}}{2} = -2 \pm i$

The area of the triangle is thus $\frac{1}{2}(5)(2) = 5$ sq. units.