Complex Numbers Q18– Practice/M (29/5/23)

Find arg $\{-sin\left(\frac{\pi}{3}\right)+icos\left(\frac{\pi}{3}\right)\}$

Solution

Approach 1

$-sin\left(\frac{\pi}{3}\right)+icos\left(\frac{\pi}{3}\right)=sin\left(-\frac{\pi}{3}\right)+icos\left(-\frac{\pi}{3}\right)$

[note that it helps to keep the angle the same in both terms]

$=cos\left(\frac{\pi}{2}-[-\frac{\pi}{3}]\right)+isin\left(\frac{\pi}{2}-[-\frac{\pi}{3}]\right)=\cos\left(\frac{5\pi}{6}\right)+isin(\frac{5\pi}{6})$

So arg $\{-sin\left(\frac{\pi}{3}\right)+icos\left(\frac{\pi}{3}\right)\}=\frac{5\pi}{6}$

Approach 2

$-sin\left(\frac{\pi}{3}\right)+icos\left(\frac{\pi}{3}\right)=-\cos\left(\frac{\pi}{2}-\frac{\pi}{3}\right)+isin\left(\frac{\pi}{2}-\frac{\pi}{3}\right)$

$=-cos\left(\frac{\pi}{6}\right)+isin\left(\frac{\pi}{6}\right)=-\{cos\left(\frac{\pi}{6}\right)-isin\left(\frac{\pi}{6}\right)\}$

Then $arg\left\{cos\left(\frac{\pi}{6}\right)-isin\left(\frac{\pi}{6}\right)\right\}=-\frac{\pi}{6}$

[as $cos\left(\frac{\pi}{6}\right)-isin\left(\frac{\pi}{6}\right)$ is the conjugate of $cos\left(\frac{\pi}{6}\right)+isin\left(\frac{\pi}{6}\right)$;

also $cos\left(\frac{\pi}{6}\right)-isin\left(\frac{\pi}{6}\right)=cos\left(-\frac{\pi}{6}\right)+isin\left(-\frac{\pi}{6}\right)$],

and so arg $\left[-\left\{cos\left(\frac{\pi}{6}\right)-isin\left(\frac{\pi}{6}\right)\right\}\right]=-\frac{\pi}{6}+\pi=\frac{5\pi}{6}$

[since multiplication by -1 is a rotation by π in the Argand diagram]

Approach 3

$$\arg\left\{-sin\left(\frac{\pi}{3}\right)+icos\left(\frac{\pi}{3}\right)\right\}=\arg\left\{i(cos\left(\frac{\pi}{3}\right)+isin\left(\frac{\pi}{3}\right))\right\}$$

$$=arg(i)+\frac{\pi}{3}=\frac{\pi}{2}+\frac{\pi}{3}=\frac{5\pi}{6}$$