Complex Numbers Overview (16/6/23)

Q1 [Practice/E]

Represent the following on the Argand diagram:

(i) $|z-i|>|z+1|$ [5 marks]

(ii) $|z-i|=2|z+1|$ [6 marks]

Q2 [6 marks]

$1+3i$ is a root of the equation $z^3+pz+q=0$ (where p & q are real). Find the other roots, and the values of p & q.

Q3 [5 marks]

Find the square roots of $3-4i$

Q4 [4 marks]

Find the solutions of $z^2=i$

Q5 [7 marks]

Find $(1+i)^{10}$

Q6 [4 marks]

Find the equation of the line satisfying

$|z+10|=|z-6-4i\sqrt{2}|$

Q7 [5 marks]

Express $(1-i)^6$ in the form $x+iy$

Q8 [8 marks]

Find the cube roots of -8 in cartesian form

Q9 [Practice/Y2/H]

Consider two roots of $z^n = cos\theta + isin\theta$:

$z_r = \cos\left(\frac{\theta}{n} + \frac{2\pi r}{n}\right) + isin\left(\frac{\theta}{n} + \frac{2\pi r}{n}\right)$

and $z_R = \cos\left(\frac{\theta}{n} + \frac{2\pi R}{n}\right) + isin\left(\frac{\theta}{n} + \frac{2\pi R}{n}\right)$

(i) Find the condition on n for z_R to equal -z_r for some R, and find R in terms of r.

(ii) Find the condition on θ for z_R to be the conjugate of z_r for some R, and find R in terms of r.

Q10 [7 marks]

Let $z = \frac{a+i}{1+ai}$. If $argz = -\frac{\pi}{4}$, find the possible values of a

Q11 [8 marks]

Find the modulus and argument of $e^{\frac{7\pi i}{10}} - e^{-\frac{9\pi i}{10}}$

Q12 [Practice/E]

For each of the following numbers, say whether they are imaginary or complex (or both):

(i) 1 (ii) i (iii) 0 (iv) $1+i$

Q13 [Practice/E]

Are these statements true or false? (Give an explanation, or a counter example, as appropriate.)

(i) All imaginary numbers are complex numbers.

(ii) All complex numbers are imaginary numbers.

(iii) All real numbers are complex numbers.

(iv) Zero is an imaginary number.

(v) The imaginary part of a complex number is an imaginary number.

(vi) All complex numbers are either real numbers or imaginary numbers.

(vii) Two imaginary numbers added together can sometimes give a real number.

(viii) If two complex numbers multiply to give a real number, then they must be conjugates of each other.

(ix) The square root of a non-real complex number is never real.

Q14 [Practice/E]

(i) How are the complex numbers z and zi related to each other geometrically?

(ii) How are the complex numbers z and $\frac{1}{z}$ related to each other geometrically?

Q15 [Practice/E]

Find $(2+5i) \div (1+3i)$ by (a) equating real and imaginary parts, and (b) another method

Q16 [Practice/E]

Solve the equation $(2+i)z + 3 = 0$ by (a) equating real and imaginary parts, and (b) another method

Q17 [Practice/E]

Solve the equation $z^2 - 2z + 2 = 0$

(a) by completing the square
(b) by equating real & imaginary parts

Q18 [Practice/M]

Find arg $\{-sin\left(\frac{\pi}{3}\right) + icos\left(\frac{\pi}{3}\right)\}$

Q19 [Practice/M]

How are the complex numbers $cos\theta + isin\theta$ and $sin\theta + icos\theta$ related?

Q20 [Practice/E]

Given that $2 - i$ is a root of the equation

$z^4 - 6z^3 - 2z^2 + 50z - 75 = 0$, find the other roots.

Q21 [Practice/M]

(i) Show geometrically that

$|z_1 + z_2| \leq |z_1| + |z_2|$

When is there equality?

(ii) Show geometrically, and also from (i) that

$|z_1 - z_2| \geq |z_1| - |z_2|$

When is there equality?

Q22 [Practice/M]

Points representing the 3 roots of the equation

$z^3 + z^2 - 7z - 15 = 0$ are plotted on an Argand diagram.

Given that one of the roots is an integer, find the area of the triangle that has these 3 points as its vertices.

Q23 [Practice/M]

Find the square roots of $-5 - 12i$

Q24 [Problem/H]

Use complex numbers to show that $sin\left(\frac{5\pi}{12}\right) = \frac{\sqrt{2}+\sqrt{6}}{4}$

Q25 [Problem/H]

Referring to the diagram, use complex numbers to prove that the diagonal OC of the rhombus OACB bisects the angle OAB.