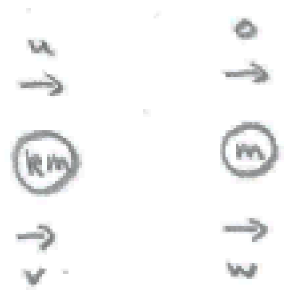
Collisions - Problem (Solution) (3 pages; 3/3/2014)

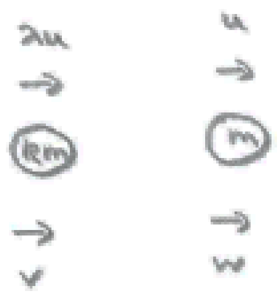
Case A

Find a condition involving k & e for v to be < 0



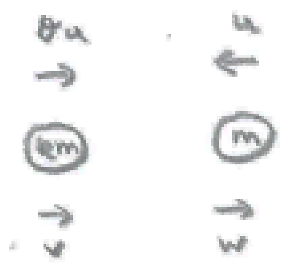
Case B

Find conditions involving k, λ & e for v to be < 0



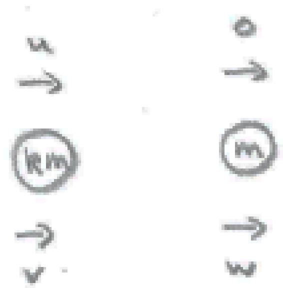
Case C

Find conditions involving k, θ & e for v to be < 0



Case A

$CoM \Rightarrow ku = kv + w\ (1) \quad \& \quad NLI \Rightarrow w - v = eu\ \ (2)$



$(1) - (2) \Rightarrow (k+1)v = (k-e)u$

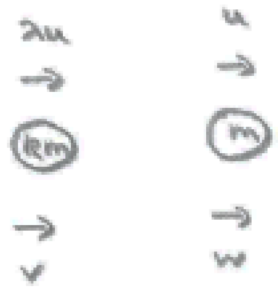
$\Rightarrow v = \frac{(k-e)u}{k+1}$; so $\boldsymbol{v < 0 \Leftrightarrow k < e}$

Case B

$CoM \Rightarrow \lambda ku + u = kv + w\ (1) \quad \& \quad NLI \Rightarrow w - v = e(\lambda - 1)u\ \ (2)$

$(1) - (2) \Rightarrow (k+1)v = u(\lambda k + 1 - e\lambda + e)$

$\Rightarrow v = \frac{u(\lambda k + 1 - e\lambda + e)}{k+1}$



So $v < 0 \Leftrightarrow \lambda k + 1 - e\lambda + e < 0$

$\Leftrightarrow \lambda(k-e) < -(1+e)$

If $k < e$, then $\lambda > \frac{1+e}{e-k}$

If $k > e$, then $\lambda < -\frac{1+e}{k-e} < 0$ (not possible)

If $k = e$, then $0 < -(1+e)$ (not possible)

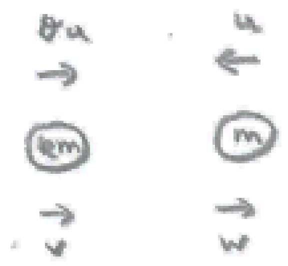
So $\boldsymbol{v < 0 \Leftrightarrow k < e}$ and $\boldsymbol{\lambda > \frac{1+e}{e-k}}$

Case C

$CoM \Rightarrow \theta ku - u = kv + w\ (1) \quad \& \quad NLI \Rightarrow w - v = e(\theta + 1)u\ \ (2)$

$(1) - (2) \Rightarrow (k+1)v = u(\theta k - 1 - e\theta - e)$

$\Rightarrow v = \frac{u(\theta k - 1 - e\theta - e)}{k+1}$



So $v < 0 \Leftrightarrow \theta k - 1 - e\theta - e < 0$

$\Leftrightarrow \theta(k - e) < (1 + e)$

If $k < e$, then $\theta > \frac{1+e}{k-e}$ (always true)

If $k > e$, then $\theta < \frac{1+e}{k-e}$

If $k = e$, then $0 < (1 + e)$ (always true)

So $\boldsymbol{v < 0 \Leftrightarrow}$ either $\boldsymbol{k \le e}$ or $\boldsymbol{\theta < \frac{1+e}{k-e}}$