

Circular Motion Exercises (Solutions) (2 pages; 11/3/17)

(1) Find the height above the earth's surface of a satellite in geostationary orbit (above the equator), using the following data:

radius of earth = 6370 km

mass of earth $\approx 6 \times 10^{24}$ kg

$G \approx 7 \times 10^{-11}$

Gravitational force = $\frac{GMm}{r^2}$

Solution

Steps:

(i) Set up $F = ma$

(ii) Establish ω

(iii) Solve for r

$$\frac{GMm}{r^2} = m\omega^2 r,$$

where M is the mass of the Earth, m is the mass of the satellite, r is the distance of the satellite from the Earth's centre, and ω is its angular speed

$$\omega = \frac{2\pi}{24 \times 3600} \text{ rads}^{-1}$$

$$\text{Hence } r^3 = \frac{GM}{\omega^2} \approx \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times (24 \times 3600)^2}{(2\pi)^2} = 7.5421 \times 10^{22}$$

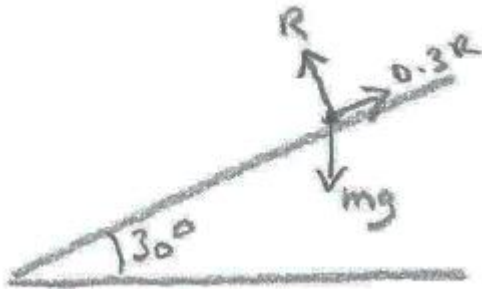
and $r = 4.22504 \times 10^7 \text{ m}$; ie 42250 km

Thus, height above Earth's surface is

$$42250 - 6380 = 35900 \text{ km (2sf)}$$

(2) A bike is being ridden round a circular track of radius 50m, banked at 30° . If the coefficient of friction is 0.3, what is the slowest speed possible?

Solution



Vertical equilibrium \Rightarrow

$$R\cos 30^\circ + 0.3R\sin 30^\circ = mg \quad (1)$$

$$\text{Circular motion} \Rightarrow R\sin 30^\circ - 0.3R\cos 30^\circ = \frac{mv^2}{50} \quad (2)$$

$$(1)\&(2) \Rightarrow R = \frac{mg}{\cos 30^\circ + 0.3\sin 30^\circ} = \frac{mv^2}{50(\sin 30^\circ - 0.3\cos 30^\circ)}$$

$$\Rightarrow v^2 = \frac{9.8(50)(0.5 - 0.3\left(\frac{\sqrt{3}}{2}\right))}{\frac{\sqrt{3}}{2} + 0.3(0.5)} = \frac{117.694}{1.01603} = 115.837$$

$$\Rightarrow v = 10.762 = 10.8 \text{ ms}^{-1} \text{ (3sf)}$$