

Boolean Algebra - Exercises (2 pages; 16/7/15)

Demonstrate the following, using the rules of Boolean algebra (see below)

$$(1) a \vee \sim [(\sim b \vee a) \wedge b] = 1$$

$$(2) (a \vee b) \wedge (a \vee \sim b) = a$$

$$(3) \{a \vee (\sim b \wedge c)\} \wedge b = a \wedge b$$

$$(4) [a \wedge (b \vee c)] \vee \sim [\sim a \vee (b \wedge c)] = a \quad [\text{MEI, D2, June 2013, Q1}]$$

$$(5) (a \wedge b) \vee (\sim a \wedge \sim b) = (\sim a \vee b) \wedge (a \vee \sim b)$$

[MEI, D2, June 2008, Q1]

$$(6) \text{ Show that } [(p \Rightarrow q) \wedge (\sim p \Rightarrow r)] \wedge \sim r \Rightarrow q$$

[MEI, D2, June 2007, Q1]

(7) Given that the statements $(X \vee \sim Y) \Rightarrow Z$ and $\sim Z$ are both true, show that Y is true. [MEI, D2, June 2012, Q1]

(8) Show that $[\sim (X \vee \sim Y) \vee Z] \wedge (\sim Z) \Rightarrow Y$ [related to (7)]

Rules of Boolean algebra (See also "Logic: Implication")

Identity: $p \wedge 1 = p$; $p \vee 1 = 1$; $p \wedge 0 = 0$; $p \vee 0 = p$

Associative: $(p \vee q) \vee r = p \vee (q \vee r)$; $(p \wedge q) \wedge r = p \wedge (q \wedge r)$

Commutative: $p \vee q = q \vee p$; $p \wedge q = q \wedge p$

Complement: $\sim(\sim p) = p$; $p \vee (\sim p) = 1$; $p \wedge (\sim p) = 0$

Distributive: $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

$$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$$

De Morgan: $\sim(p \vee q) = (\sim p) \wedge (\sim q)$; $\sim(p \wedge q) = (\sim p) \vee (\sim q)$

Absorption: $p \wedge p = p$; $p \vee p = p$;

$$p \wedge (p \vee q) = p; p \vee (p \wedge q) = p$$