Numerical Solution of Equations - Bisection Method

(2 pages; 22/10/18)

(1) Change of Sign

If the location of a root of $f(x) = 0$ is known approximately (say X), then an interval estimate for the true root α can be obtained as (X_1, X_2), where $X_1 = X - \delta_1$ and $X_2 = X + \delta_2$, if $f(X_1) < 0$ and $f(X_2) > 0$ (or vice-versa); δ_1 and δ_2 being small numbers.

[Note: The change of sign idea is also used for other numerical methods of solving equations.]

(2) Bisection Method

Step 1: Write equation in the form $f(x) = 0$ [e.g. $x^3 - x - 1 = 0$]

Step 2: Use the change of sign idea to establish a small interval containing one of the roots [e.g. $f(1) = -1$ & $f(2) = 5$]

Step 3: Find the sign at the mid-point of the interval [$f(1.5) = 0.875$]

Step 4: Obtain a reduced interval [(1, 1.5)] containing a change of sign

Then repeat the process.

x_1	x_2	$f(x_1)$	$f(x_2)$	$x_m = \frac{1}{2}(x_1 + x_2)$	$f(x_m)$
1	2	-1	5	1.5	0.875
1	1.5	-1	0.875	1.25	-0.29688
1.25	1.5	-0.29688	0.875	1.375	0.22461
1.25	1.375	-0.29688	0.22461	1.3125	-0.05151
1.3125	1.375	-0.05151	0.22461	1.34375	0.08261
1.3125	1.34375	-0.05151	0.08261	1.328125	

(3) Notes

(i) Provided suitable starting points are chosen, the Bisection method will find a root - unless the root is repeated.

(ii) However, there may be more than one root in an interval, and one drawback of the method is that these additional roots may be overlooked.

(iii) The Bisection method automatically gives an interval estimate for the root (ie there is an indication of the accuracy of the root obtained).

(iv) But the number of decimal points involved can become unwieldy.