Arithmetic Series – Q4 [Problem/E] (17/6/21)

For an arithmetic sequence with 1st term a and common difference d, show that the sum of the 1st n terms is

$\frac{n}{2}[2a + (n-1)d]$ by starting with $\sum_{k=1}^{n}[a + (k-1)d]$

Solution

$$\sum_{k=1}^{n}[a + (k-1)d] = \left[(a-d)\sum_{k=1}^{n} 1\right] + d\sum_{k=1}^{n} k$$

$$= (a-d)n + d \cdot \frac{1}{2}n(n+1)$$

$$= \frac{n}{2}(2a - 2d + dn + d) = \frac{n}{2}[2a + (n-1)d]$$