Arithmetic Sequences & Series - Exercises

(2 pages; 23/1/17)

[*/** indicates harder exercises]

(1) If teams A, B, C, D & E in some sporting competition have to play each other once, how many games are there in total?

(2)(**) Extend (1) to find a formula for $1 + 2 + 3 + \cdots + n$

[See **Solutions** for Ideas]

(3) For each of the following arithmetic sequences, find an expression for a_k:

(a) in the form $a_k = p + q(k - 1)$

(b) in the form $a_k = mk + c$

(c) in the form $a_k = a_{k-1} + t$; $a_1 = u$ $(k \geq 2)$

(where p, q, m, c, t & u are to be found)

(i) $4, 7, 10, 13, 16, \ldots$

(ii) $-2, -1, 0, 1, 2, \ldots$

(iii) $8, 6, 4, 2, 0, \ldots$

(4) If $a_3 = 7$ and $a_{10} = 42$ are terms in an arithmetic sequence, find an expression for a_k.

(5)(*) Find

(i) $\sum_{k=1}^{20}(2k+3)$

(ii) $\sum_{k=1}^{40}(10-4k)$

(6)(*) Solve the equation $\sum_{k=1}^{n}(100-5k)=0$

(7) For each of the arithmetic sequences in (5), find $\sum_{k=1}^{100} a_k$

(8) If I pay £50 into a bank account, then £60 a year later, followed by £70 the following year, and so on, increasing by £10 each year,

(i) How long will it take for the amount I pay in each year to reach £200?

(ii) How long will it take for the amount in the bank account to reach £1000?

(9)(*) For an arithmetic sequence with 1st term a and common difference d, show that the sum of the 1st n terms is

$\frac{n}{2}[2a+(n-1)d]$ by starting with $\sum_{k=1}^{n}[a+(k-1)d]$