Algebraic and Transcendental numbers (1 page; 5/6/23)

An irrational number is one that cannot be written as a fraction (where the numerator and denominator are integers).

Irrational numbers can be broken down further into surds and transcendental numbers.

In order to define these two types of numbers, we need to define an 'algebraic number': one which is the solution of a polynomial equation with rational coefficients (in which case, it will be the solution of a polynomial equation with integer coefficients, after multiplying through by the LCM of the denominators).

Surds are then defined as irrational algebraic numbers, whilst transcendental numbers are those irrational numbers that are not algebraic numbers.

Surds include any combination of roots such as $\sqrt{2}$, $\sqrt[3]{5}$ etc.

Transcendental numbers include:

π and e (2.718281828459045235360287 4...)

[note the strange coincidence in the repetition of the sequence 1828 in e]

$sin a, cos a$ etc., where a is an algebraic number other than zero

a^b, where a is an algebraic number, but not equal to zero or 1, and b is an irrational algebraic number (eg $2^{\sqrt{2}}$ is transcendental).