Vectors Q25 (3/7/23)

(i) Find the plane containing the points
$(3,0,-1), (5,2,-3)$ and $(4,2,4)$, in parametric form

(ii) Hence find the equation of the plane in Cartesian form.

Solution

(i) A general point on the plane is

$$\underline{r} = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} + \lambda[\begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}] + \mu[\begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}]$$

ie $\underline{r} = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$

(ii) We can write

$x = 3 + 2\lambda + \mu$

$y = 2\lambda + 2\mu$

$z = -1 - 2\lambda + 5\mu$

Using the eq'n for x to eliminate μ:

$y = 2\lambda + 2(x - 3 - 2\lambda)$ or $y = -2\lambda + 2x - 6$

$z = -1 - 2\lambda + 5(x - 3 - 2\lambda)$ or $z = -12\lambda + 5x - 16$

Then, eliminating λ:

$-12\lambda = 6y - 12x + 36$ & also $z - 5x + 16$,

so that $6y - 12x + 36 = z - 5x + 16$

Hence $-7x + 6y - z = -20$ or $7x - 6y + z = 20$

[To check that this plane contains the given points:

$7(3) - 6(0) + (-1) = 20$,

$7(5) - 6(2) + (-3) = 20$

& $7(4) - 6(2) + 4 = 20$]