

Vectors Q21 (3/7/23)

Use the vector product to find the area of the triangle with corners A (1,2,3), B (4,5,6) & C (9,8,7)

Solution

$$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} \text{ \& } \overrightarrow{AC} = \begin{pmatrix} 8 \\ 6 \\ 4 \end{pmatrix}$$

$$\text{Area} = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2} \left| \begin{array}{ccc} i & 3 & 8 \\ j & 3 & 6 \\ k & 3 & 4 \end{array} \right|$$

$$= \frac{1}{2} |-6i + 12j - 6k|$$

$$= \frac{1}{2} \times 6 \times |i - 2j + k| = 3 \times \sqrt{1 + 4 + 1} = 3\sqrt{6} \text{ units}^2$$