Vectors Q20 (3/7/23)
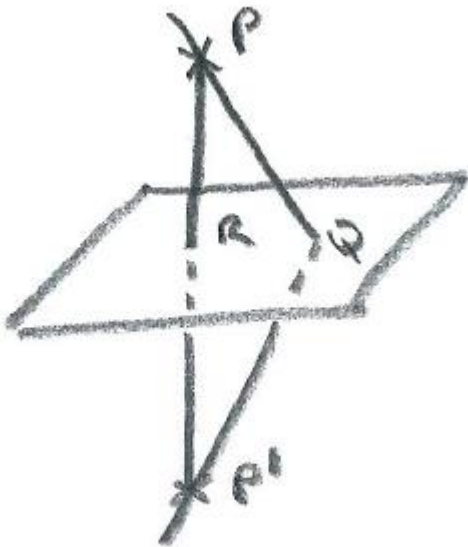
Find the reflection of the line $\frac{x-2}{3} = \frac{y+4}{1}; z = 3$ in the plane $y = 4$

Solution

Let P be $\begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix}$, say.

Q is intersection of the line and plane :

Line is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$

Substituting into the eq'n of the plane: $-4 + \lambda = 4 \Rightarrow \lambda = 8$

So Q is $\begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix} + 8 \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 26 \\ 4 \\ 3 \end{pmatrix}$

Line PR is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

R is intersection of PR and the plane:

$-4 + \mu = 4 \Rightarrow \mu = 8$

So P' is $\begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix} + 2(8) \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 12 \\ 3 \end{pmatrix}$

Eq'n of P'Q is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 12 \\ 3 \end{pmatrix} + \theta[\begin{pmatrix} 26 \\ 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 12 \\ 3 \end{pmatrix}]$

ie $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 12 \\ 3 \end{pmatrix} + \theta\begin{pmatrix} 24 \\ -8 \\ 0 \end{pmatrix}$, or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 12 \\ 3 \end{pmatrix} + \theta'\begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$

or $\dfrac{x-2}{3} = \dfrac{y-12}{-1}; z = 3$