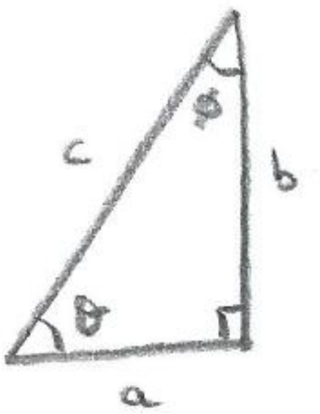
Trigonometry - Relations (3 pages; 15/4/21)

(1) Symmetry

(i) $\cos(-\theta) = cos\theta$ & $\sin(-\theta) = -sin\theta$ (from the graphs)

(ii) $\sin(180° - \theta) = sin\theta$ and $\cos(360° - \theta) = cos\theta$, by the symmetries of the sine and cosine graphs about $\theta = 90°$ and 180°, respectively.

(2) Complementary angles



$$\sin\theta = \frac{b}{c} = \cos\phi$$

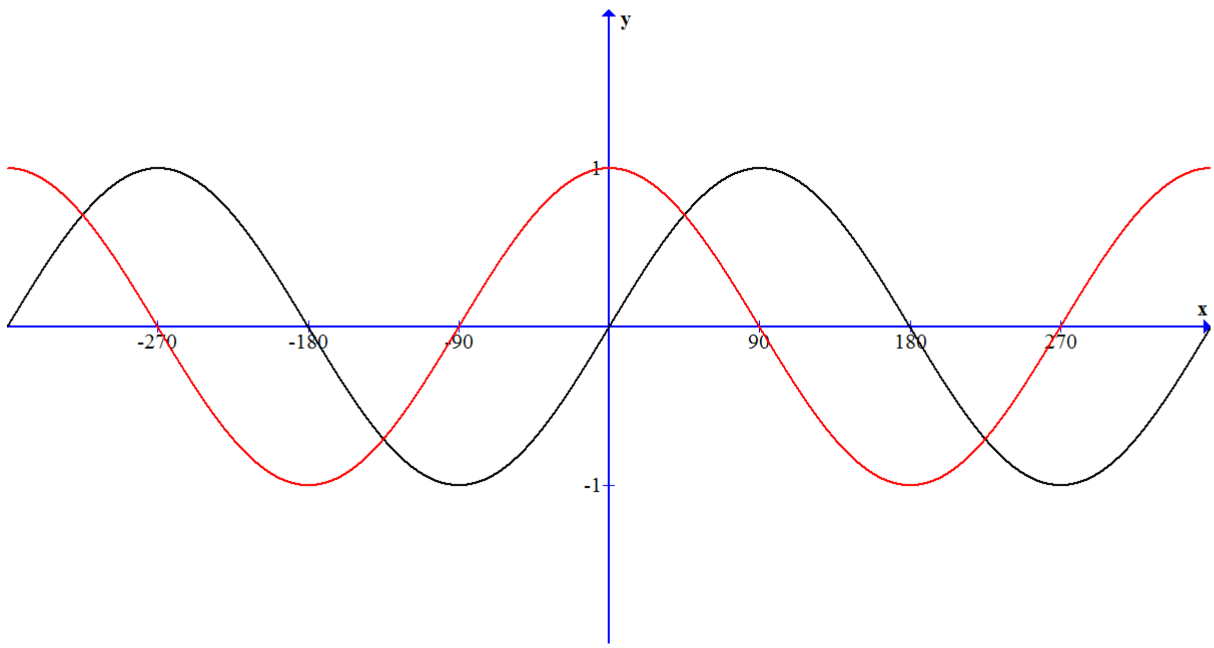
$$\cos\theta = \frac{a}{c} = \sin\phi$$

$sin\theta = cos\phi = \cos\,(90° - \theta)$ and $cos\theta = sin\phi = sin\,(90° - \theta)$

θ & ϕ are 'complementary' angles (ie they add up to 90°). This is the origin of the term 'cosine': $cos\theta$ is the sine of the angle complementary to θ.

Similarly, $cot\theta = tan\phi$, and $cosec\theta = \frac{1}{sin\theta} = \frac{1}{cos\phi} = sec\phi$

(3) Translations



Graphs of $y = sinx\ (black)$ & $y = cosx\ (red)$

[Note that replacing θ with $\theta + \alpha$ produces a translation of α to the left, and replacing θ with $\theta - \alpha$ produces a translation of α to the right. See "Transformations of Functions" for further details.]

As an alternative to using the compound angle formulae (see Part 2), the following examples can be tackled by considering the translation and/or reflection involved; or often just by examining the graph.

Examples

(i) $\sin(360° - \theta) = \sin(-\theta) = -sin\theta$ (or from the graph)

(ii) $\sin(\theta + 180°) = \sin(\theta - 180°) = -\sin(180° - \theta) = -sin\theta$

(or from the graph, or by noting that replacing θ in $sin\theta$ by

$\theta + 180°$ produces a translation of $180°$ to the left, which gives the graph of $-sin\theta$)

(iii) $\cos(180° - \theta) = \cos\,(\theta - 180°)$, which is obtained from $cos\theta$

by a translation of 180° to the right, which is seen to be $-cos\theta$

(as can be verified from the compound angle formula).

(iv) $sin(\theta + 90°)$ can be obtained from $sin\theta$ by a translation of 90° to the left, which is seen to be $cos\theta$