Trigonometry – Solution of equations (2 pages; 15/4/21)

(1) Solution of simple equations - for a range of angles

Example: To solve $\sin\left(2\theta - \frac{\pi}{6}\right) = 0.5$ for $0 < \theta \leq 2\pi$,

let $\phi = 2\theta - \frac{\pi}{6}$, so that $-\frac{\pi}{6} < \phi \leq 4\pi - \frac{\pi}{6}$

Then $\phi = \frac{\pi}{6}$, $\pi - \frac{\pi}{6}$, and also $\frac{\pi}{6} + 2\pi$ & $\pi - \frac{\pi}{6} + 2\pi$

and values for θ are obtained from $\theta = \frac{1}{2}(\phi + \frac{\pi}{6})$

[When $cos\phi = 0.5$, $\phi = \frac{\pi}{3}$, $2\pi - \frac{\pi}{3}$, and multiples of 2π can be added or subtracted.]

(2) General solution of simple equations

The approach above (for a limited range of θ) can be applied, to obtain two ('base') solutions in one cycle (except for $sin\theta = 1 \; or -1$). Then the general solutions can be derived as follows:

(i) For eg $sin\theta = 0.5$, the base solutions are $\theta = \frac{\pi}{6}$, $\pi - \frac{\pi}{6}$

A more concise alternative to $\frac{\pi}{6} + 2k\pi \;\; or \;\; \frac{5\pi}{6} + 2k\pi$ is to note that the solutions lie alternately $\frac{\pi}{6}$ ahead of and behind the multiples of π, so that we can write $\theta = n\pi + (-1)^n \left(\frac{\pi}{6}\right)$

So the general solution for $y = sinx$ is $x = n\pi + (-1)^n arcsiny$

(ii) For eg $cos\theta = 0.5$, the base solutions are $\theta = \frac{\pi}{3}$, $2\pi - \frac{\pi}{3}$, or alternatively $\theta = \frac{\pi}{3}$ & $-\frac{\pi}{3}$, from which we obtain the general solution of $\theta = 2k\pi \pm \frac{\pi}{3}$

So the general solution for $y = cosx$ is $x = 2k\pi \pm arccosy$

(iii) For eg $tan\theta = 1$, the general solution is just $\theta = \frac{\pi}{4} + k\pi$

So the general solution for $y = tanx$ is $x = arctany + k\pi$