Trigonometry Q4 (30/6/23)

Solve the equation $sinx - cosx = 0.5,\ for\ 0° < x < 360°$

Solution

Method 1

Write $sinx - cosx = Rsin(x - \alpha) = R(sinxcos\alpha - cosxsin\alpha)$,

so that $Rcos\alpha = 1$ & $Rsin\alpha = 1$,

and hence $R^2(cos^2\alpha + sin^2\alpha) = 2$, so that $R = \sqrt{2}$

Also $tan\alpha = 1$, so that $\alpha = 45°$ (for example).

Thus the original equation becomes $\sqrt{2}sin(x - 45°) = 0.5$

Then let $u = x - 45°$, so that $-45° < u < 315°$

$sinu = \frac{1}{2\sqrt{2}} \Rightarrow u = 20.70481\ or\ 180 - 20.70481$

(and there are no other solutions within the range for u)

So $x = u + 45° = 65.7°\ or\ 204.3°$ (1dp)

Method 2

$sinx - cosx = 0.5 \Rightarrow tanx - 1 = 0.5secx$

$\Rightarrow (tanx - 1)^2 = \frac{sec^2x}{4}$,

if we exclude solutions of $tanx - 1 = -0.5secx$

$\Rightarrow 4(tan^2x - 2tanx + 1) = 1 + tan^2x$

$\Rightarrow 3tan^2x - 8tanx + 3 = 0$

$\Rightarrow tanx = \frac{8\pm\sqrt{28}}{6} = \frac{1}{3}(4 \pm \sqrt{7}) = 2.21525\ or\ 0.45142$

$\Rightarrow x = 65.7°\ or\ 24.3°$,

as well as $65.7 + 180 = 245.7°$ and $24.3 + 180 = 204.3°$

But $24.3°$ and $245.7°$ are solutions of $tanx - 1 = -0.5secx$ and can therefore be excluded.

Thus the solutions are $x = 65.7°\ or\ 204.3°$

Method 3

$sinx - cosx = 0.5 \Rightarrow sin^2x = (cosx + 0.5)^2$

but this will include solutions of $-sinx - cosx = 0.5$, which will need to be removed

$\Rightarrow 1 - cos^2x = cos^2x + cosx + \frac{1}{4}$

$\Rightarrow 2cos^2x + cosx - \frac{3}{4} = 0$

$\Rightarrow 8cos^2x + 4cosx - 3 = 0$

$\Rightarrow cosx = \frac{-4\pm\sqrt{16+96}}{16} = \frac{-1\pm\sqrt{7}}{4} = -0.91144$ or 0.41144

$\Rightarrow x = 155.7°, 360 - 155.7 = 204.3°, 65.7°$

$or\ 360 - 65.7 = 294.3°$

The only solutions of the required equation are

$x = 65.7°\ and\ 204.3°$

(the other two are found to be solutions of $-sinx - cosx = 0.5$)

Method 4

$t = tan\left(\frac{x}{2}\right) \Rightarrow cosx = \frac{1-t^2}{1+t^2}$ & $sinx = \frac{2t}{1+t^2}$ (standard results - see "Trigonometry - Part 2")

Then, substituting into our equation:

$\frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2} = \frac{1}{2}$

$\Rightarrow 2\{2t - (1 - t^2)\} = 1 + t^2 \quad \Rightarrow t^2 + 4t - 3 = 0$

$\Rightarrow t = \frac{-4 \pm \sqrt{28}}{2} = -2 \pm \sqrt{7} = 0.64575 \; or - 4.64575$

$\Rightarrow \frac{x}{2} = 32.852° \; or - 77.852° + 180°$

(these are the only values between $0°$ *and* $180°$, which is the permissible range for $\frac{x}{2}$)

and hence $x = 65.7° \; or \; 204.3°$ (1dp)