Trigonometry Q3 (30/6/23)

By means of the substitution $t = tan\left(\frac{x}{2}\right)$, solve the equation $secx + tanx = 1$, where $0° \leq x \leq 360°$.

Solution

$t = \tan\left(\frac{x}{2}\right) \Rightarrow \sec x = \frac{1+t^2}{1-t^2}$ & $\tan x = \frac{2t}{1-t^2}$

Then, substituting into the equation:

$\frac{1+t^2}{1-t^2} + \frac{2t}{1-t^2} = 1$ (*)

$\Rightarrow 1 + t^2 + 2t = 1 - t^2 \Rightarrow 2t^2 + 2t = 0$

$\Rightarrow t = 0 \; or -1$

But if $t = -1$, $\sec x$ is undefined.

So $\frac{x}{2} = 0 \; or \; 180$ (as $\frac{x}{2}$ must lie between $0°$ or $180°$ (incl.)),

and hence $x = 0° \; or \; 360°$.