TMUA Exercises – Transformations - Sol'ns

(9 pages; 4/11/22)

(1) Reflection in the line $x = L$: What composite transformation is equivalent to this?

Solution

translation of $\begin{pmatrix}-L\\0\end{pmatrix}$, followed by reflection in y-axis, followed by

translation of $\begin{pmatrix}L\\0\end{pmatrix}$

$y = f(x) \rightarrow ?$

translation of $\begin{pmatrix}-L\\0\end{pmatrix}$: $y = f(x) \rightarrow y = f(x + L)$;

reflection in y-axis: $y = f(x + L) \rightarrow y = f(-x + L)$

translation of $\begin{pmatrix}L\\0\end{pmatrix}$: $y = f(-x + L) \rightarrow y = f(-[x - L] + L)$

$= f(2L - x)$

(2) What combination of transformations converts $y = 2^x$ to $y = 2^{4x-2}$?

Solution

$y = 2^x \to y = 2^{4x}$ is a stretch of scale factor $\frac{1}{4}$ in the x-direction

Then $y = 2^{4x} \to y = 2^{4(x-\frac{1}{2})} = 2^{4x-2}$ is a translation of $\begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}$

[Alternatively, $y = 2^{4x} \to y = \left(\frac{1}{4}\right) 2^{4x} = 2^{4x-2}$ is a stretch of scale factor $\frac{1}{4}$ in the y-direction.]

(3) (i) Find a series of transformations that can be applied to $y = \frac{1}{x}$ to produce $y = \frac{3x-2}{6x-1}$.

(ii) Hence or otherwise, sketch the curve $y = \frac{3x-2}{6x-1}$.

Solution

(i) $\frac{3x-2}{6x-1} = \frac{3x-\frac{1}{2}-\frac{3}{2}}{6x-1} = \frac{1}{2} - \frac{3}{12}(\frac{1}{x-\frac{1}{6}})$

So a possible series of transformations is:

a translation of $\begin{pmatrix} \frac{1}{6} \\ 0 \end{pmatrix}$,

followed by a stretch of scale factor $\frac{1}{4}$ in the y-direction,

followed by a reflection in the x-axis,

followed by a translation of $\begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix}$

[Note: $\frac{1}{2} - \frac{3}{12}\left(\frac{1}{x-\frac{1}{6}}\right) = \frac{1}{2} - \frac{1}{4x-\frac{2}{3}}$, so an alternative series of transformations is:

a translation of $\begin{pmatrix} \frac{2}{3} \\ 0 \end{pmatrix}$ [$\frac{1}{x} \to \frac{1}{x-\frac{2}{3}}$] followed by

a stretch of scale factor $\frac{1}{4}$ in the x-direction [$\frac{1}{x-\frac{2}{3}} \to \frac{1}{4x-\frac{2}{3}}$], followed by a reflection in the x-axis, followed by a translation of $\begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix}$.

Alternatively, $\frac{1}{4x-\frac{2}{3}}$ could be obtained instead by a stretch of scale factor $\frac{1}{4}$ in the x-direction [$\frac{1}{x} \to \frac{1}{4x}$] (or a stretch of scale factor $\frac{1}{4}$ in the y-direction

[$\frac{1}{x} \to \frac{1}{4}(\frac{1}{x})$]), followed by a translation of $\begin{pmatrix} \frac{1}{6} \\ 0 \end{pmatrix}$ [$\frac{1}{4x} \to \frac{1}{4(x-\frac{1}{6})}$].]

(ii) As an alternative to performing the transformations in (i):

Step 1: $x = 0 \Rightarrow y = 2$; $y = 0 \Rightarrow x = \frac{2}{3}$

Step 2: vertical asymptote when $6x - 1 = 0 \Rightarrow x = \frac{1}{6}$

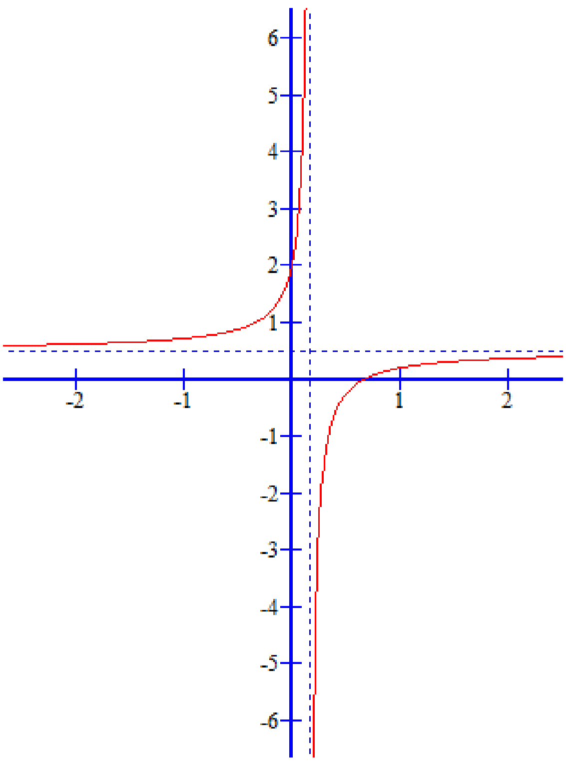
$x = \frac{1}{6} + \delta$ ($\delta > 0$ is small) $\Rightarrow y = \frac{3x-2}{6x-1} = \frac{-}{+}$; ie $y < 0$

[$x = \frac{1}{6} - \delta \Rightarrow y = \frac{-}{-}$; ie $y > 0$]

Step 3: $\lim_{x\to\infty} \frac{3x-2}{6x-1} = \lim_{x\to\infty} \frac{3-\frac{2}{x}}{6-\frac{1}{x}} = \frac{3}{6} = \frac{1}{2}$ (and also as $x \to -\infty$)

Step 4: When $x = 100$, $y = \frac{298}{599} < \frac{1}{2}$, so that $y \to \frac{1}{2}^-$ as $x \to \infty$

and when $x = -100$, $y = \frac{-302}{-601} > \frac{1}{2}$, so that $y \to \frac{1}{2}^+$ as $x \to -\infty$



(4) What combination of transformations converts $y = 3^{-x}$ to $y = 3^{2x-1}$?

Solution

$y = 3^{-x} \to y = 3^x$ is a reflection in the y-axis.

$y = 3^x \to y = 3^{x-1}$ is a translation of $\binom{1}{0}$

$y = 3^{x-1} \to y = 3^{2x-1}$ is a stretch of scale factor $\frac{1}{2}$ in the

x-direction