TMUA Exercises – Polynomials - Sol'ns

(10 pages; 3/11/22)

(1) Factorise $2x^3 - 33x^2 - 6x + 11$

Solution

If the factorisation is of the form $= (x + a)(2x^2 + bx + c)$,

then a must be $\pm$ a factor of 11

Applying the factor theorem, this is found not to be the case.

Let $2x^3 - 33x^2 - 6x + 11 = (2x + a)(x^2 + bx + c)$,

Equating coefficients gives:

$-33 = 2b + a, -6 = 2c + ab$ & $11 = ac$

Testing the possible combinations of a & c ($\pm$ the factors of 11) shows that $a = -1, c = -11$ & $b = -16$

ie $2x^3 - 33x^2 - 6x + 11 = (2x - 1)(x^2 - 16x - 11)$.

(2) Find the minimum value of $(x^2 - 4x + 3)(x^2 + 4x + 3)$

Solution

$(x^2-4x+3)(x^2+4x+3)=(x-3)(x-1)(x+3)(x+1)$

$=(x^2-9)(x^2-1)$

$=x^4-10x^2+9$

$=(x^2-5)^2-16$

which has -16 as its minimum value

(3) How many solutions are there to $x^3 - 6x^2 + 9x + 2 = 0$?

Solution

$\Leftrightarrow x(x^2 - 6x + 9) = -2$

$\Leftrightarrow x(x - 3)^2 = -2$

So one solution, from graph of $y = x(x - 3)^2$

(4) Write out the possible factorisations of $x^n - y^n$ and $x^n + y^n$

Solution

$x^n - y^n = (x - y)(x^{n-1} + x^{n-2}y + \cdots + xy^{n-2} + y^{n-1})$

or $(x + y)(x^{n-1} - x^{n-2}y + \cdots + xy^{n-2} - y^{n-1})$ if n is even

$x^n + y^n = (x + y)(x^{n-1} - x^{n-2}y + \cdots - xy^{n-2} + y^{n-1})$ if n is odd

(5) Factorise $2x^3 - 33x^2 - 6x + 11$

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ie $2x^3 - 33x^2 - 6x + 11 = (2x - 1)(x^2 - 16x - 11)$.