TMUA Exercises – Logarithms - Sol'ns (7 pages; 4/11/22)

(1) (i) Show that $log_2 3 > \frac{3}{2}$

(ii) Find an upper bound for $log_2 3$ (as small as possible)

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Solution

$log_2 3 > \frac{3}{2} \Leftrightarrow 3 > 2^{\frac{3}{2}}$ (as $y = 2^x$ is an increasing function)

$\Leftrightarrow 3^2 > 2^3$

(ii) Find an upper bound for $log_2 3$ (as small as possible)

Solution

Suppose that $log_2 3 < \frac{m}{n}$

Then $3 < 2^{(\frac{m}{n})}$ and $3^n < 2^m$

As $243 = 3^5 < 2^8 = 256$, $log_2 3 < \frac{8}{5}$

[and $\frac{8}{5}$ is a reasonably low upper bound, as 243 & 256 are reasonably close]

(2) Prove that $log_b c = \frac{log_a c}{log_a b}$

Solution

rtp $log_a b \, log_b c = log_a c$ (*)

Method 1

Let $b = a^x$ & $c = b^y$

Then $c = (a^x)^y = a^{xy}$

and $log_a c = xy = log_a b \, log_b c$, as required

Method 2

(*) is equivalent to $a^{log_a b \, log_b c} = a^{log_a c}$ (as $y = a^x$ is an increasing function)

ie $(a^{log_a b})^{log_b c} = c$ (**)

and the LHS equals $b^{log_b c} = c$, so that (**) holds, and hence (*) holds also

(3) Show that $\log\left(4-\sqrt{15}\right) = -\log\left(4+\sqrt{15}\right)$

Solution

$$\log\left(4-\sqrt{15}\right) = -log\left(\frac{1}{4-\sqrt{15}}\right) = -log\left(\frac{4+\sqrt{15}}{16-15}\right) = -\log\,(4+\sqrt{15})$$