TMUA Exercises – General - Sol'ns (4 pages; 3/11/22)

(1) (i) Does $\sqrt{4}$ equal 2 or ± 2? (ii) Simplify $\sqrt{x^2}$

Solution

(i) $\sqrt{4} = 2$

[By convention, the square root symbol denotes the positive root (consider the $\pm$ in the quadratic formula, which wouldn't be needed if the square root symbol covered both positive and negative roots). Note that the solution of $x^2 = 4$ is $x = \pm\sqrt{4}$.]

(ii) $\sqrt{x^2} = |x|$ [Note that x could be negative.]

(2) Find the square roots of $49 - 12\sqrt{5}$

Solution

Let $x^2 = 49 - 12\sqrt{5}$

Consider $x = a + b\sqrt{5}$

Then $a^2 + 2ab\sqrt{5} + 5b^2 = 49 - 12\sqrt{5}$

Let $a^2 + 5b^2 = 49 \ \& \ 2ab = -12$

[a variation on Equating Coefficients]

Looking for integer solutions, we see that either

$a = 2$ & $b = -3$ or $a = -2$ & $b = 3$ work.