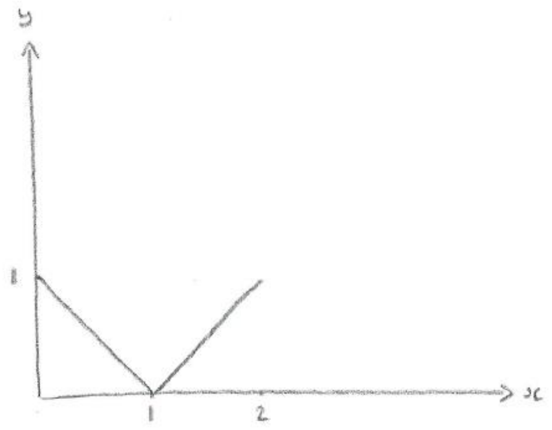
TMUA Exercises – Curve Sketching - Sol'ns

(6 pages; 4/11/22)

(1) Sketch the graph of $\sqrt{x^2 - 2x + 1}$ for $0 \leq x \leq 2$

Solution

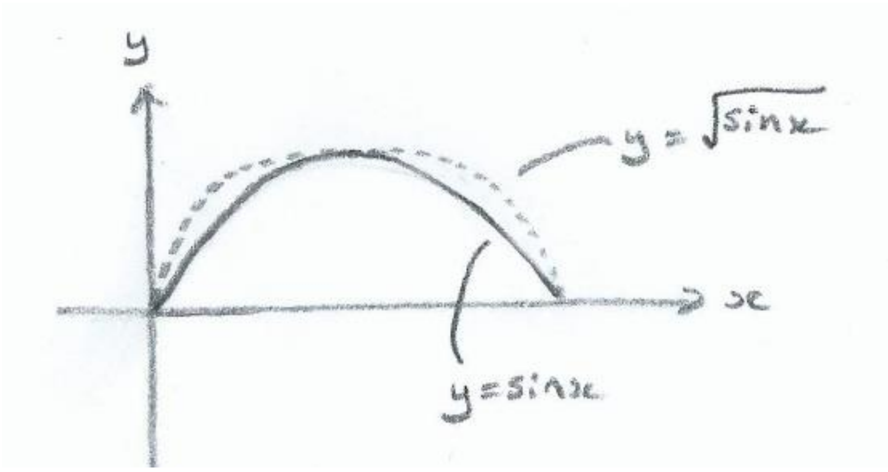


For $0 \leq x \leq 1, \sqrt{x^2 - 2x + 1} = \sqrt{(x-1)^2} = \sqrt{(1-x)^2} = 1 - x$

For $1 \leq x \leq 2, \sqrt{x^2 - 2x + 1} = \sqrt{(x-1)^2} = x - 1$

(2) Sketch (i) $y = \sqrt{sinx}$ and (ii) $y = (sinx)^{\frac{1}{n}}$ for large positive integer n (for $0 \leq x \leq \pi$ in both cases).

Solution



(i) Note that, for $0 < y < 1, \sqrt{y} > y$

So, for $y = \sqrt{sinx}$, the graph will hug the $y - axis$ more than for $y = sinx$.

Also, if $f(x) = \sqrt{sinx}$, $f'(x) = \frac{1}{2}(sinx)^{-\frac{1}{2}} cosx$,

so that $f'(0) = \infty$ (strictly speaking, it is 'undefined');

ie the graph is vertical at $x = 0$ (and also $x = \pi$, by symmetry).

(ii) The effect is greater for larger n, and the graph tends to a rectangular shape.

(3) Sketch the curve $x^2 = y(1 - y)$

Solution

$y(1-y) = -(y^2 - y) = -\left(y - \frac{1}{2}\right)^2 + \frac{1}{4}$

So curve is $x^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{4}$

ie a circle centre $(0, \frac{1}{2})$ and radius $\frac{1}{2}$