TMUA Specimen Paper 2 Solutions (9 pages; 25/10/23)

Q1

$2x^2 + 2y^2 - 8x + 12y + 15 = 0$

$\Leftrightarrow 2(x-2)^2 - 8 + 2(y+3)^2 - 18 + 15 = 0$

$\Leftrightarrow 2(x-2)^2 + 2(y+3)^2 = 11$ or $(x-2)^2 + (y+3)^2 = \frac{11}{2}$

so that the radius is $\sqrt{\frac{11}{2}}$

Answer is B

Q2

$y = \frac{(3x-2)^2}{x\sqrt{x}} = (3x-2)^2 x^{-\frac{3}{2}}$

$\Rightarrow \frac{dy}{dx} = 2(3x-2)(3)x^{-\frac{3}{2}} + (3x-2)^2(-\frac{3}{2})x^{-\frac{5}{2}}$

When $x = 2, \frac{dy}{dx} = 2(4)(3)\left(\frac{1}{2\sqrt{2}}\right) + 16(-\frac{3}{2})\left(\frac{1}{4\sqrt{2}}\right)$

$= \left(\frac{1}{\sqrt{2}}\right)(12-6) = 3\sqrt{2}$

Answer is B

Q3

Step 1 is liable to introduce a spurious sol'n (as the eq'n

$-\sqrt{x+5} = x+3$ also leads to $x + 5 = x^2 + 6x + 9$ [it remains to be seen though whether $-\sqrt{x+5} = x+3$ has any sol'ns]

We can see that $x = -4$ is not a sol'n of the original eq'n, but that $x = -1$ is. So statements A and B are both Incorrect. And statement C is correct (and statements D and E are Incorrect).

Answer is C

Q4

Answer is A (not that hard!)

Q5

Approach 1:

$2^5 \approx 3^3 \Rightarrow log_3(2^5) \approx 3$

$\Rightarrow 5log_3 2 \approx 3$, so that $log_3 2 \approx \frac{3}{5}$

Approach 2: Suppose that $log_3 2$ is approximately $\frac{a}{b}$
This is equivalent to $2 \approx 3^{\frac{a}{b}}$, and hence $2^b \approx 3^a$

So, writing $a = 3, b = 5$

[as $3^3 = 27$ is reasonably close to $2^5 = 32$]
gives $log_3 2 \approx \frac{3}{5}$

Answer is A

Q6

Maximum height is $\frac{5649\dot{9}}{79.5}\ cm$; ie effectively $\frac{5650}{79.5}\ cm$ (though arguably not correct!)

Answer is F

Q7

$x = 0 \Rightarrow y^3 = 1 \Rightarrow y = 1 \Rightarrow B\ or\ C$

$y = 0 \Rightarrow x^3 = 1 \Rightarrow x = 1 \Rightarrow\ C$

Answer is C

Q8

The sum is $n + (n + 1) + (n + 2) + (n + 3) = 4n + 6$

This will only be a multiple of 6 if n is a multiple of 3, so statement C is true.

Answer is C

Q9

As an example where $(*)'$ is true (for a simplified 3 day week):

Mon: 2 MPs

Tue: no MPs

Wed : 1 MP

Only B & E are compatible with this example.

But B isn't compatible with the following example which satisfies $(*)'$:

Mon: 1 MP

Tue: no MPs

Wed : 1 MP

So answer is E

Q10

[Be careful not to misread this as $y = log_2x$]

$log_x2 = \frac{1}{log_2x}$

Answer: E

Q11

$A = tan\left(\frac{3\pi}{4} - \pi\right) = tan\left(-\frac{\pi}{4}\right) = -tan\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

$B = 2$

$C = 1$

$3 < D < 4$

$E < 0.5^{10} < 1$

Answer: D

Q12

[Remainder theorem]

Answer: A

Q13

The information can be written as

$F > G, H > L, L > G, R > G$; eg FRHLG

We can say for certain that G comes last.

The only other constraint is that $H > L$.

There are 4! ways of ordering F, R, H & L, without this constraint,

and $H > L$ for half of these.

So number of ways is 12.

Answer: C

Q14

There will be 4 solutions to $\frac{dy}{dx} = 0$, and any complex roots
will come in conjugate pairs, as the coefficients are real.
So B cannot be possible, as this would mean that there were 3 real roots and 1 complex root.
[This is technically outside the TMUA syllabus, but the official solution, not involving complex numbers, is considerably longer.]

Answer: B

Q15

Statement 1 ≡ $a \geq b$ (always true)

Statement 2 ≡ $(a-b)^2 \geq 0$ (always true)

Re. Statement 3: $a \geq b \Rightarrow ac \geq bc$ if $c \geq 0$, but not if $c < 0$

So Statement 3 is not always true.

Answer: E

Q16

$a_2 = 2 - 1 = 1$

$a_3 = 1 + 1 = 2$

$a_4 = 2 - 1 = 1$

...

$a_{99} = 1 + 1 = 2$

$a_{100} = 2 - 1 = 1$

So $\sum_{n=1}^{100} a_n = 50(2+1) = 150$

Answer: A

Q17

Answer: E

Q18

Approach 1

Let the 5 numbers be $m - a - b, m - a, m, m + c, m + c + d$

where $a, b, c \; \& \; d \; are \; all \; \geq 0$

Then $(m - a - b) + (m - a) + m + (m + c) + (m + c + d) = 0$ (1)

and $(m + c + d) - (m - a - b) = 20$ (2)

so that $c + d + a + b = 20$, and

$(m - a - b) + (m - a) + m + (m + c) + (m - a - b) + 20 = 0$

$\Rightarrow 5m = 3a + 2b - c - 20$

Thus $5m = 2(c + d + a + b) + a - 3c - 2d - 20$

$= 20 + a - 3c - 2d$

$= 20 + (20 - c - d - b) - 3c - 2d$

[aiming for a form where the letters all have negative signs]

$= 40 - 4c - 3d - b$

and this is maximised when $b = c = d = 0$, so that $a = 20$

(as $c + d + a + b = 20$) and $m = 8$

[Then the 5 numbers are $-12, -12, 8, 8, 8$]

Approach 2 (Trial & Improvement)

One sol'n (satisfying the 2 conditions) is -10, 0, 0, 0, 10,

So that M (the largest possible value for the median) ≥ 0 (as indicated by the multiple choice options).

The median can then be increased to 5 without changing the 1st and last values, giving -10, -10, 5, 5, 10 (the 4th value has to be at least 5, and if equals 5, then the 2nd value has to be -10, to maintain the mean of 0).

The median can then be increased to 8 (as suggested by the MC options) by increasing the 4th & 5th values to 8, which allows the 1st value to be lowered to -12, which accommodates a 2nd value of -12 (needed to maintain the mean of 0).

Thus, $M \geq 8$

But $M = 20$ isn't possible, as the 4th & 5th values would then have to be at least 20, forcing the 1st value to be at least 0 (for the range to be 20). But this would give a mean > 0.

So $8 \leq M < 20$, and from the MC options available, M must be 8.

Answer: E

Q19

[A graphical approach looks promising here, but unfortunately turns out to be too complicated.]

The two equations can be rewritten as:

$x^3 + ax^2 - bx - c = 0$ (1)

and $x^3 - ax^2 - bx + c = 0$ (2)

[Given that only the signs of even powers of x differ]

Let $y = -x$

Then (2) becomes $-y^3 - ay^2 + by + c = 0$

or $y^3 + ay^2 - by - c = 0$, which has the same roots as (1).

So, if (1) has one positive and two negative roots, (2) will have one negative and two positive roots.

Answer: B

Q20

[Begin by looking for something to narrow down the options.]

[Assuming P true doesn't lead immediately to anything tangible.]

Q true ⇒ S true ⇒ exactly one of $PR'T'$, $P'RT'$ & $P'R'T$ holds

Consider $PR'T'$: this is possible (an odd number of statements are true, Mr R's first name could be Rupert; both statements made by women are true)

So $QSPR'T'$ is possible.

This means that (assuming exactly one of the answers is correct), the answer must be D or H.

Consider H, where exactly 2 statements could be true:

If H holds, then P cannot be true (as 2 is not an odd number).

We have seen that Q true ⇒ 3 statements are true, so (for H to hold), Q cannot be true.

If R is true, then P is true, but this has been shown not to be the case.

So, if exactly 2 statements are true, then they must be S and T.

But if T is true, then S is not true, so exactly 2 statements is not possible.

Thus H is not possible, and **the answer must be D**.