Series – Q3: Method of Differences [5 marks] (20/6/21)

Exam Boards

OCR : Pure Core (Year 2)

MEI: Core Pure (Year 1)

AQA: Pure (Year 1)

Edx: Core Pure (Year 2)

Given that $\frac{1}{2r-1} - \frac{2}{2r+1} + \frac{1}{2r+3} = \frac{8}{(2r-1)(2r+1)(2r+3)}$, use the method of differences to show that

$\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)(2r+3)} = \frac{n(n+2)}{3(2n+1)(2n+3)}$ [5 marks]

Solution

$$\sum_{r=1}^{n}\frac{1}{(2r-1)(2r+1)(2r+3)}=\frac{1}{8}\{\sum_{r=1}^{n}\frac{1}{2r-1}-\sum_{r=1}^{n}\frac{2}{2r+1}+\sum_{r=1}^{n}\frac{1}{2r+3}\}$$

$$=\frac{1}{8}\{\frac{1}{1}+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\cdots+\frac{1}{2n-5}+\frac{1}{2n-3}+\frac{1}{2n-1}$$

$$-2(\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}+\cdots+\frac{1}{2n-3}+\frac{1}{2n-1}+\frac{1}{2n+1})$$

$$+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}+\frac{1}{11}+\cdots+\frac{1}{2n-1}+\frac{1}{2n+1}+\frac{1}{2n+3}\}$$ [2 marks]

As the highlighted items cancel: [1 mark]

$$=\frac{1}{8}\{1-\frac{1}{3}-\frac{1}{2n+1}+\frac{1}{2n+3}\}$$ [1 mark]

$$=\frac{2(2n+1)(2n+3)-3(2n+3)+3(2n+1)}{24(2n+1)(2n+3)}$$

$$=\frac{8n^2+16n}{24(2n+1)(2n+3)}$$

$$=\frac{n(n+2)}{3(2n+1)(2n+3)}$$ [1 mark]