Series – Q2: Method of Differences [3 marks] (20/6/21)

Exam Boards

OCR : Pure Core (Year 2)

MEI: Core Pure (Year 1)

AQA: Pure (Year 1)

Edx: Core Pure (Year 2)

Given that $\frac{1}{2r} - \frac{1}{2(r+2)} = \frac{1}{r(r+2)}$, use the method of differences to find $\sum_{r=1}^{n} \frac{1}{r(r+2)}$ [3 marks]

Solution

$\sum_{r=1}^{n} \frac{1}{r(r+2)} = (\sum_{r=1}^{n} \frac{1}{2r}) - \left(\sum_{r=1}^{n} \frac{1}{2(r+2)}\right)$

$= \left(\frac{1}{2} + \frac{1}{4} + [\frac{1}{6} + \frac{1}{8} + \cdots + \frac{1}{2(n-2)} + \frac{1}{2(n-1)} + \frac{1}{2n}]\right)$

$- \left([\frac{1}{6} + \frac{1}{8} \ldots + \frac{1}{2n}] + \frac{1}{(2n+2)} + \frac{1}{(2n+4)}\right)$ [1 mark]

As the terms in square brackets cancel: [1 mark]

$= \frac{1}{2} + \frac{1}{4} - \frac{1}{(2n+2)} - \frac{1}{(2n+4)}$

$= \frac{3}{4} - \frac{1}{2(n+1)} - \frac{1}{2(n+2)}$ [1 mark]

[For exam purposes, it may not be necessary to write this as a single fraction.]

$= \frac{1}{4(n+1)(n+2)}\{3(n+1)(n+2) - 2(n+2) - 2(n+1)\}$

$= \frac{1}{4(n+1)(n+2)}\{3n^2 + 9n + 6 - 2n - 4 - 2n - 2\}$

$= \frac{1}{4(n+1)(n+2)}\{3n^2 + 5n\}$

$= \frac{n(3n+5)}{4(n+1)(n+2)}$

[This can be checked by substituting $n = 1$.]