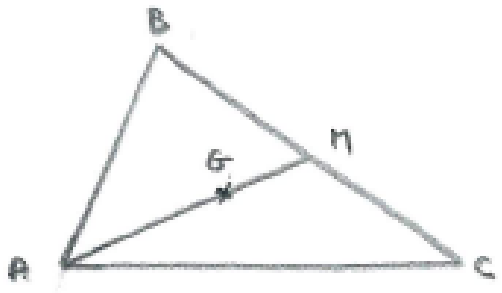
STEP/Vectors Q5 (30/6/23)

Given that the centre of mass of a triangular lamina lies 2/3 of the way along any of the medians, prove that it has position vector $\frac{1}{3}$ $(\underline{a} + \underline{b} + \underline{c})$.



Solution

$$\overrightarrow{OG} = \overrightarrow{OA} + \overrightarrow{AG}$$

$$= \underline{a} + \frac{2}{3}\overrightarrow{AM}$$

$$= \underline{a} + \frac{2}{3} . ½ \, (\overrightarrow{AB} + \overrightarrow{AC})$$

$$= \underline{a} + \frac{1}{3}[\,(\underline{b} - \underline{a}) + (\underline{c} - \underline{a})]$$

$$= \frac{1}{3}(\underline{a} + \underline{b} + \underline{c})$$

So if $\underline{a} = \begin{pmatrix} a_x \\ a_y \end{pmatrix}$ etc, $\overrightarrow{OG} = \begin{pmatrix} \frac{1}{3}(a_x + b_x + c_x) \\ \frac{1}{3}(a_y + b_y + c_y) \end{pmatrix}$